

Statistics of Magnetic Fluxes of Massive Stars and the Enigma of the Origin of Magnetic Fields in Neutron Stars

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Abstract. Statistical properties of mean magnetic fields and magnetic fluxes of normal stars are investigated based on data from the catalogue of magnetic fields by Bychkov et al. (2009) and recent measurements from the literature. As a measure of the mean field, we use the *rms* longitudinal magnetic field \mathcal{B} . We estimated the magnetic fluxes \mathcal{F} of massive stars and found that they have a log–normal distribution with the logarithm of mean flux $\log(\mathcal{F}) = 27.7$. A similar shape of the magnetic flux distribution holds for the entire neutron star population, but with a much smaller mean value of $\log(\mathcal{F}) = 24.5$ in contradiction with the hypothesis by Ferrario and Wickramasinghe (2006) that magnetic fields of neutron stars have a fossil origin, and that magnetic fluxes are conserved over the stellar evolution up to the neutron star formation. On the other hand, the magnetic flux distribution for magnetars is similar to that for OB stars, but more narrow. A special case is the millisecond pulsars with relatively small magnetic fluxes and a mean $\log(\mathcal{F}) = 19.8$. We discuss various possibilities to explain such a large difference between the magnetic fluxes of normal and neutron stars and compare the model distributions of magnetic fluxes with that, obtained from the measured stellar magnetic fields.

1 Introduction

The magnetic fields of solar–type stars seem to be formed by the dynamo–mechanism. Massive stars have no convective envelopes and the dynamo–mechanism is probably not very effective for them. These stars have strong regular magnetic fields. The hypothesis most reliable for them at the present time is a relic nature of OBA star magnetic fields (Braithwaite & Nordlund, 2006).

The formation of the magnetic fields of massive stars with masses $M > 8–10 M_{\odot}$ is connected with generation of strong magnetic fields of the neutron stars (e.g. Ruderman, 1972). A fraction of neutron stars with super strong magnetic fields about $10^{14}–10^{15}$ G is about of 1% accordingly *McGill SGR/AXP Online Catalog*.

In the paper by Ferrario & Wickramasinghe (2006) it is supposed that the magnetic field of neutron stars is a relic of a field of a massive OB star. They also supposed that the stellar magnetic flux remains constant during all the stellar evolution after the main sequence and up to the stellar collapse. Spruit (2008) supposed that the relic magnetic field can be strengthened during the collapse due to the magnetorotational instability (MRI). Moreover, in this paper it is mentioned that there can exist stable configurations of magnetic fields of neutron stars. It means that the magnetic field

remains nearly constant during the cooling of neutron stars.

2 Magnetic Fields of Normal Stars, White Dwarfs and Neutron Stars

2.1 Normal OB Stars

Our investigation of statistical properties of magnetic fields and magnetic fluxes of normal stars is based largely on data collected in the catalogue by Bychkov et al. (2009) and its previous version (Bychkov et al., 2003). In these catalogues all the measurements up to 2009 are collected. After publishing the Bychkov et al. (2009) catalogue, new measurements of magnetic fields of OB stars were made. We added the data from papers by Bouret et al. (2008), Hubrig (2008, 2009a, 2009b), Kholtygin et al. (2007), Petit et al. (2008), Schnerr et al. (2008) and McSwain (2008) which was not included in the catalogue by Bychkov et al. (2009) in our statistical analysis.

Magnetic field measurements give us mainly the value of \overline{B}_l , which is an average over the whole visible stellar disk longitudinal projection of the stellar magnetic field (*effective magnetic field*). It means that the value of \overline{B}_l depends on the rotational phase and changes from some minimal value B_{min} to the maximal value B_{max} , which may have different signs. The effective magnetic field \overline{B}_l can not be used for the statistical investigations of magnetic fields of the stellar ensemble. It means that we need to use some kind of a global field feature, which can be calculated from the measured field values. Further we will use the *rms* field (see, e. g. Bohlender et al., 1993):

$$\mathcal{B} = \sqrt{\frac{1}{n} \sum_{i=1}^n (\overline{B}_l^i)^2}, \quad (1)$$

where i is the number of magnetic field measurements and all the measured values of effective magnetic fields are summed, n is the number of observations. Kholtygin et al. (2010a) have shown that in the case of a dipole field configuration, the *rms* field weakly depends on random values of rotational phases ϕ , the inclination angle i and the angle between β rotational axis and the axis of the magnetic dipole.

As a measure of the field measurement accuracy we use the value

$$\Sigma_{\mathcal{B}} = \sqrt{\frac{1}{n} \sum_{i=1}^n (\sigma_{B^i})^2}. \quad (2)$$

Here σ_{B^i} is the *rms* error of a single field measurement with number i . The commonly used value of $\sigma_{\mathcal{B}}$ is not a standard deviation of the *rms* magnetic field \mathcal{B} , however, it can be used as a measure of field measurement error.

We suppose that if

$$\mathcal{B} > 2 \Sigma_{\mathcal{B}}, \quad (3)$$

then the field measured is real. Figure 1 demonstrates the mean magnetic field values averaged over different spectral classes we obtained.

For the stars of F and later spectral classes, the values presented in Fig. 1 are only the upper limits of the mean magnetic field due to the presence of a large number of the small-scale local magnetic fields on the stellar surface. The contribution of these local fields to the global magnetic field of a star is small because of the reduction effects.

First of all we see a large rise in the mean magnetic field between the O and B spectral classes according to the conclusion by Kholtygin et al. (2010a, 2010b). If we suppose that the magnetic fluxes for O and B stars are close, we can explain this difference by larger radii of O stars. This

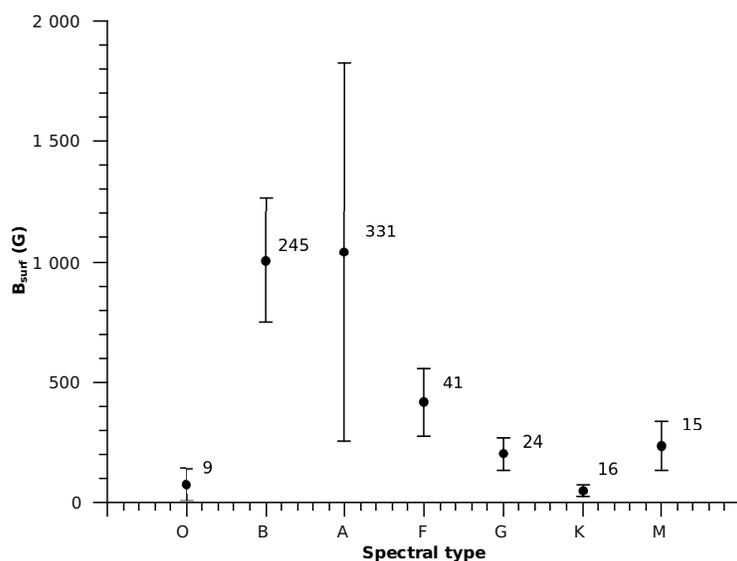


Figure 1: Magnetic fields of normal stars averaged over different spectral classes. The numbers mark the number of stars with measured magnetic fields according to the criteria (3).

effect can be also connected with a loss of the magnetic flux of O stars due to their powerful stellar winds.

2.2 White Dwarfs

The surface fields of white dwarfs B_s are adopted from the catalogue by Nalezyty & Madej (2004). The value of B_s can be determined (see, for example, Ferrario and Wickramasinghe, 2006) as:

$$B_s = \frac{\mathcal{F}}{\pi R_*^2}, \quad (4)$$

where R_* is a stellar radius and \mathcal{F} is a stellar magnetic flux at the level of photosphere. The values of \mathcal{F} can be calculated via the relation (9).

It means that

$$\mathcal{B} \approx B_s/4. \quad (5)$$

2.3 Neutron Stars

Magnetic fields of neutron stars (NS) were taken from the *ATNF pulsar catalogue* (Manchester et al., 2005) and from *McGill SGR/AXP Online Catalog* (<http://www.physics.mcgill.ca/~pulsar/magnetar/main.html>). In these sources the surface magnetic fields B_s are presented. These values are estimated from the position of the cyclotron lines in the spectra of NS, or via the standard relation (e. g., Kaspi, 2010):

$$B_s = 3.2 \cdot 10^{19} \sqrt{P\dot{P}}, \quad (6)$$

where P is the rotation period of a pulsar in seconds, $\dot{P} = dP/dt$ is the first derivative of P . The connection between the *rms* magnetic field \mathcal{B} and B_s is described by the formula (5).

3 Magnetic Fluxes

3.1 Main Relations

Full magnetic flux \mathcal{F} at the level of the stellar photosphere in the spherical coordinate system (θ, φ) can be found as

$$\mathcal{F} = \iint_S (\mathbf{B} \cdot \mathbf{n}) dS = R_*^2 \int_{-\pi}^{\pi} \int_0^{2\pi} B_r \sin \theta d\theta d\varphi. \quad (7)$$

Here \mathbf{B} is a vector of magnetic induction, B_r is its radial projection, \mathbf{n} is a vector of a normal to the stellar surface, and R is the stellar radius. For the case of dipole field from the relation (7) we have:

$$\mathcal{F}_d = (4/3)\pi B_p R_*^2. \quad (8)$$

where B_p is the polar field. To estimate the magnetic fluxes of stars with the known values of \mathcal{B} , we will use the following relation:

$$\mathcal{F} = \pi B_s R_*^2 \approx 4\pi \mathcal{B} R_*^2, \quad (9)$$

This formula gives a good estimate of the flux as even in the case of a dipole field $\mathcal{F} \approx 5/3 \mathcal{F}_d$. For more complex field configurations, a difference between the exact value of \mathcal{F} , calculated via the relation (7) and the estimation (9) is smaller.

3.2 Magnetic Fluxes of Massive OB Stars

We calculate the magnetic fluxes of all stars presented in the catalogue by Bychkov et al. (2003), Bychkov et al. (2009) and for additional objects the magnetic fields of which are given in the papers cited in Section 2.1. For the evaluation of stellar radii we used the data from the *Catalogue of Apparent Diameters and Absolute Radii of Stars, CADARS* (Pasinetti Fracassini et al., 2001). The radii of stars which are not in the catalogue were calculated using the standard relation between the spectral and luminosity classes of a star and its radius (Cox, 2000).

3.3 Magnetic Fluxes of Neutron Stars

3.3.1 Radii of Neutron Stars

To estimate the radii of neutron stars we use the mass–radius relation (e.g. Steiner et al., 2010; Zhang et al., 2007; Zhang, 2009). According to Zhang (2009) the radius of a neutron star is

$$R_6 = 1.27 m^{1/3} \left(\frac{A}{0.7} \right)^{-2/3}. \quad (10)$$

Here $R_6 = R/(10 \text{ km})$, where R is the radius of a star, $m = M/M_\odot$ is the mass of the star in solar masses, parameter $A = (m/R_6^3)^{1/2}$.

The radii of 4 neutron stars were calculated from the relation (10) using the values of the parameter A , which are given by Zhang (2009). For all the other stars we assume that their radii are equal to the standard for neutron stars value of $R = 10 \text{ km}$.

3.4 Dependence of Magnetic Fields on the Stellar Radii

We calculate the magnetic fluxes for all massive OB stars with known magnetic fields, for white dwarfs, the surface magnetic fields B_s of which are given in a catalogue by Należyty & Madej (2004) and for all known neutron stars. The magnetic flux distribution functions for OB and neutron stars

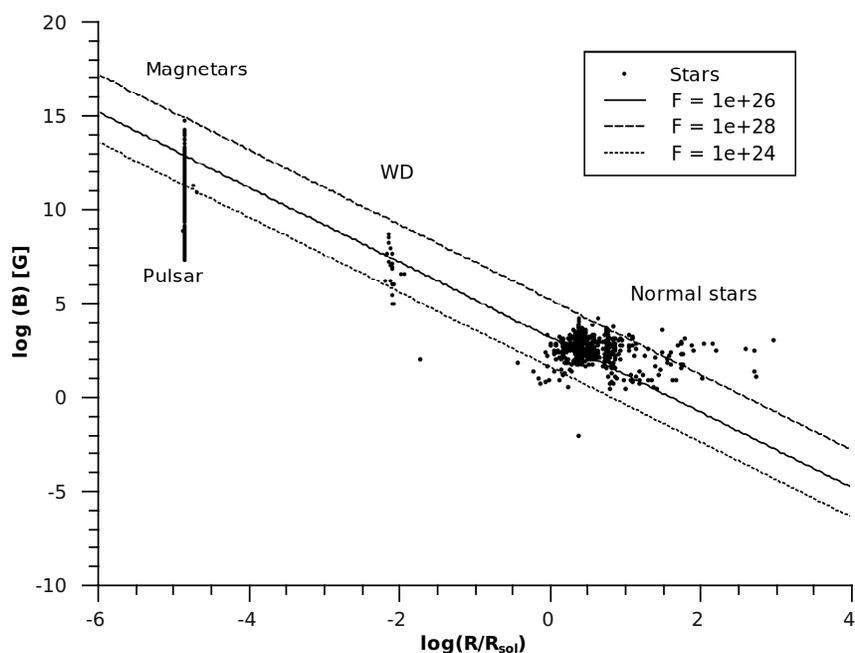


Figure 2: Stellar magnetic fields \mathcal{B} vs. their radii. We mark the positions of normal stars in the catalogue (Bychkov et al., 2009), white dwarfs and neutron stars (points). The dashed lines are the lines of constant magnetic fluxes: 10^{24} , 10^{26} and 10^{28} G·cm² (from bottom to top).

are discussed in the next section. Here we consider the dependence of the stellar magnetic field on their radii. This dependence is presented in Fig. 2.

From the analysis of Fig. 2 we can see that the magnetic fluxes of white dwarfs and normal stars are mainly in the interval $\mathcal{F} \in [10^{25}, 10^{28}]$. At the same time, there is a large number of neutron stars having the magnetic fluxes much smaller than the lower limit $\log \mathcal{F} = 24$ of the magnetic flux variation interval for the normal stars and white dwarfs.

3.5 Magnetic Flux Distribution Function

We determine the differential distribution function of magnetic fluxes $f(\mathcal{F})$ as follows:

$$N(\mathcal{F}, \mathcal{F} + \Delta\mathcal{F}) \approx N f(\mathcal{F}) \Delta\mathcal{F}, \quad (11)$$

where $N(\mathcal{F}, \mathcal{F} + \Delta\mathcal{F})$ is the number of stars with fluxes in the interval $(\mathcal{F}, \mathcal{F} + \Delta\mathcal{F})$, and N is a full number of stars with measured flux values.

Our study shows that the magnetic flux distribution is often asymmetric due to the lack of stars with small magnetic fluxes. At the same time, the behavior of the flux distribution for large fluxes $\mathcal{F} > \bar{\mathcal{F}}$, where $\bar{\mathcal{F}}$ is a mean flux for the considered group of stars, and is more regular. We suppose that in the region $\mathcal{F} > \bar{\mathcal{F}}$ the distribution function $f(\mathcal{F})$ can be described by a log-normal law.

Let us introduce the designation $x = \log(\mathcal{F})$. In the case of the log-normal approximation of the magnetic flux distribution, the distribution function for the value of x in the region $x > \bar{x} = \log(\bar{\mathcal{F}})$ obeys the normal law:

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} \cdot e^{-\frac{(x-\bar{x})^2}{2\sigma^2}}, \quad (12)$$

where σ designates a standard deviation for the normal distribution. The value of σ is the half-width of the distribution (12) in dex.

3.5.1 Chi-Square Criteria

To test the goodness of our fit of the magnetic flux distribution to the log-normal law, we use the standard Pearson's chi-square statistics (Brandt, 1975). Taking into account the fact that we fit the distribution only for values $x_i > \bar{x}$, we can determine the chi-square statistics by the usual manner:

$$\tilde{\chi}^2 = \sum_{i=0}^n \frac{(m_i - Np_i^o)^2}{Np_i^o}, \quad (13)$$

where $N > n$ is the total number of determinations of the magnetic fluxes, p_i^o is a probability that the value of x is in the interval (x_i, x_{i+1}) on the condition that the distribution of the x value is described by the law (12). It is obvious that the value of

$$p_i^o = \int_{x_i}^{x_{i+1}} f(x) dx. \quad (14)$$

The probability density function for the value of $z = \tilde{\chi}^2$ in the limit $N \rightarrow \infty$ accordingly by Taylor (1997) is

$$g(z) = \frac{1}{2^{r/2} \Gamma(\frac{r}{2})} z^{\frac{r}{2}-1} e^{-\frac{1}{2}z}, \quad (15)$$

where $\Gamma(\alpha)$ in expression (15) is the common Gamma-function (Janke et al., 1960). The value of the degree of freedom in this expression is $r = n - 2$ in our case.

For testing the reliability of the hypothesis that the distribution (12) can be used to approximate the magnetic flux distribution function we calculate the value of $\tilde{\chi}^2$ and find a level of false alarm probability (FAP) α , where

$$P_r(\chi^2 \geq \tilde{\chi}^2) = \int_{\tilde{\chi}^2}^{\infty} g(z) dz \leq \alpha. \quad (16)$$

In this case the law (12) can be accepted at the FAP level α .

3.5.2 Magnetic Flux Distribution for Massive and Neutron Stars

Using the relation (9) we calculate the magnetic fluxes for all the stars with measured magnetic fields. The distribution of magnetic fluxes for massive stars with masses $M > 8 M_{\odot}$ is given in Fig. 3. We approximate this distribution with the log-normal law. The mean value of magnetic flux for OB stars is $\log \mathcal{F} = 27.7$, and a standard deviation is $\sigma = 0.64$ dex.

It should be mentioned that the ensemble of known pulsars is a mixture of the neutron stars of the different nature with strongly different magnetic fields. Figure 4 presents the distributions of magnetic fluxes for various kinds of neutron stars. We can see on the figure that the shapes of the distribution functions for normal pulsars and massive OB stars are very close, whereas the mean fluxes for these objects are strongly different (see the Table 1). The mean fluxes of pulsars are about three orders of magnitude lower than those for OB stars. The mean fluxes of millisecond radio pulsars appeared to be still less by two or three orders of magnitude. And only the mean magnetic fluxes of magnetars are close to the fluxes of massive stars.

In Table 1 we give the mean fluxes of OB stars and various sorts of neutron star magnetic fluxes and widths of the flux distribution (2nd–3rd columns). In the 4th column the numbers of stars with known magnetic fluxes are presented. In the next two columns with numbers 5 and 6 we present the parameters of the approximations of magnetic flux distribution using only the right side of the distribution function (see subsection 3.5 for details). The last two columns contain the value of $\tilde{\chi}^2$, determined via a relation (13) and the FAP level for the fits. We can see from the table that the quality of fit by the log-normal fit is good only for the normal pulsars.

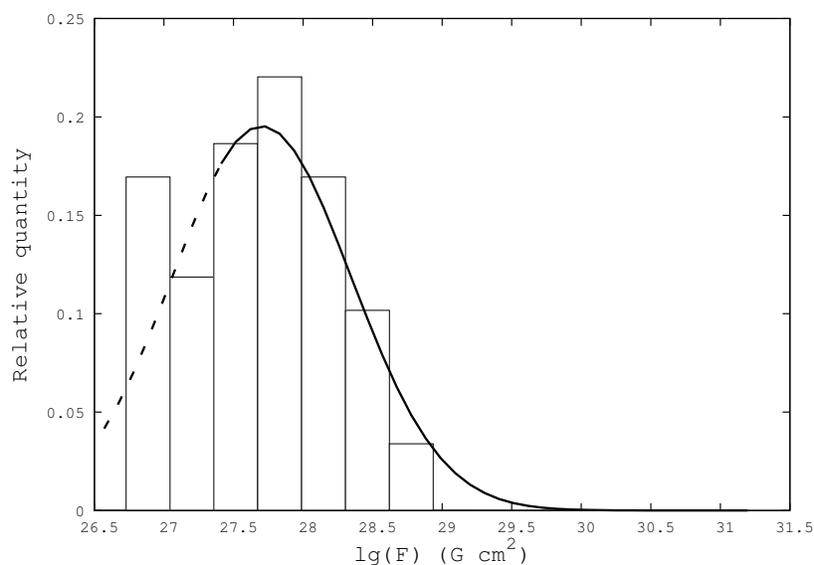


Figure 3: Magnetic flux distribution for massive OB stars. Solid line shows the approximation of the distribution by a log-normal law in the region $\mathcal{F} \geq \bar{\mathcal{F}}$. The dashed line is an extrapolation of this dependence for the lower values of \mathcal{F} .

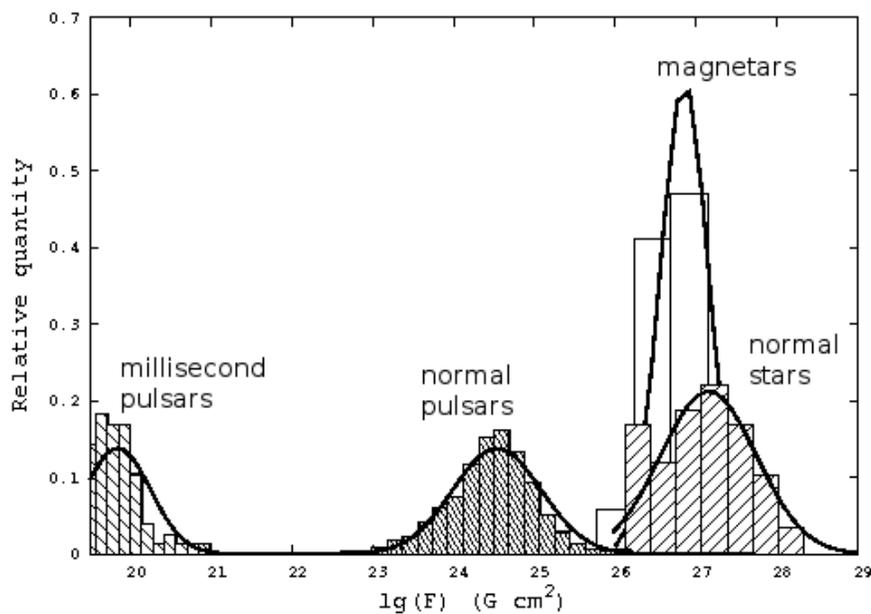


Figure 4: Magnetic flux distribution for massive OB stars in a comparison with those for millisecond pulsars, normal pulsars and magnetars. The thick line shows the approximations of these distributions by the log-normal law (12) in the region $\log(\bar{\mathcal{F}}) > \bar{x} = \log(\mathcal{F})$ and the extrapolation of this dependence for the lower values of magnetic fluxes.

Table 1: Parameters of the log-normal approximation of the flux distribution for massive OB stars and various types of neutron stars

Objects	Mean over an ensemble			Right Side Fit			
	$\overline{\log \mathcal{F}}$	σ	N_{stars}	$\overline{\log \mathcal{F}}$	σ	χ^2	FAP
$M > 8 M_{\odot}$	27.69	0.53	59	27.76	0.64	3.40	0.15
Millisecond Pulsars	19.76	0.35	77	19.40	0.57	5.87	0.17
Normal Pulsars	24.46	0.54	1573	24.73	0.46	—	0.01
Magnetars	26.71	0.32	17	26.95	0.28	0.02	$3E-5$

4 Discussion of Results

4.1 Origin of the Magnetic Field of Neutron Stars

Massive OB stars have a strong and regular surface magnetic field. The nature of their fields remains not very clear up to date. The leading possibility is the “fossil field” theory (Cowling, 1945). The fossil field hypothesis is supported by some observations such as very high strengths of the field in some stars, an apparent stationary state of the field and the wide range of field strength magnitudes observed.

Massive OB stars are the progenitors of neutron stars. A simple idea to explain the origin of strong magnetic fields of neutron stars is the compression of magnetic flux which is already present in the progenitor stars. Woltjer (1964) was the first who predicted the magnetic field value of 10^{14} – 10^{16} G for the supernova stellar remnants, still before neutron stars had been discovered.

Many authors (e. g., Ruderman, 1972; Ferrario & Wickramasinghe, 2006) have pointed out that the distribution of magnetic fluxes in magnetic A and B stars, white dwarfs, and neutron stars is very similar. This fact supports the hypothesis that the stellar magnetic fluxes are generated on or even before the main sequence stage, and are then inherited by the stellar remnants (white dwarfs or neutron stars).

Alternatively, Thompson & Duncan (1993) have suggested that a dynamo process operates in newborn neutron stars. They predicted fields up to 10^{15} – 10^{16} G in neutron stars with few-millisecond initial periods, and suggested that such fields could explain the peculiarities of magnetars. Some magnetars were later confirmed to spin down at a rate consistent with a strong dipole field 10^{14} – 10^{15} G (Woods, 1999).

Recently Endeve et al. (2010) explored the capacity of the stationary accretion shock instability to generate magnetic fields by adding a weak, stationary and radial magnetic field to an initially spherically symmetric fluid configuration. They found that this mechanism can lead to an increase in the total magnetic energy by about two orders of magnitude. This stationary accretion shock instability may contribute to a neutron star magnetization.

The relic and dynamo origins of the magnetic field of neutron stars are not mutually exclusive. A strong field might be present in the collapsing star, but later become deformed and amplified by some combination of convection, differential rotation, and magnetic instabilities (e. g., Spruit, 2002). For both mechanisms, the field and its supporting currents are not likely to be confined to the solid crust of the neutron star, but distributed in most of the stellar interior, which is mainly a fluid mixture of neutrons, protons, electrons, and possibly more exotic particles.

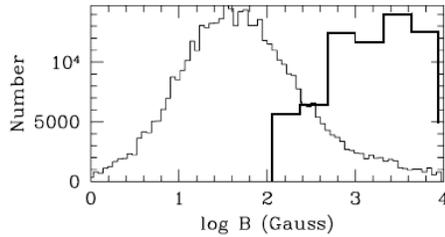


Figure 5: A comparison with a model by Ferrario & Wickramasinghe (2006). The real surface magnetic field distribution function for massive stars (right thick line) and predicted field distribution (left thin line) are normalized on the total number (447000) of objects.

4.2 Comparison with Models

The relic hypothesis is often connected with a hypothesis (e. g., Ferrario & Wickramasinghe, 2006) that stellar magnetic fluxes are stable during the entire stellar evolution up to their final phases. Our comparison of the mean magnetic fluxes of different populations of neutron stars and massive OB stars (see Table 1) contradicts this hypothesis.

In a paper by Ferrario & Wickramasinghe (2006) a synthetic model of neutron stars was constructed. The authors supposed that the magnetic flux of a massive star is conserved during its evolution to neutron stars. For the parameterized models of the distribution of magnetic flux on the main sequence and of the birth spin period of neutron stars, they calculated the expected properties of isolated radio pulsars in the Galaxy using the initial mass function and star formation rate as a function of the Galactocentric radius. The 1374-MHz *Parkes Multi-Beam Survey* (Faulkner et al., 2004) of isolated radio pulsars was used to constrain the parameters of the model and to deduce the required distribution of magnetic fields on the main sequence.

They found an agreement with observations for a model with star formation rate that corresponds to a supernovae rate of 2 per century in the Galaxy, originating from stars with masses in the range of $8 \leq M/M_{\odot} \leq 45$, and predicted about 447 000 active pulsars in the Galaxy with luminosities greater than 0.19 mJy kpc^2 .

The model by Ferrario & Wickramasinghe (2006) was used to predict the field distribution of the progenitor OB stars. It appears that the predicted distribution peaks at the surface field $B_s = 46 \text{ G}$. Moreover only 8 % of massive stars have fields exceeding 1 kG. The higher-field progenitors yield a population of 24 neutron stars with fields in excess of 10^{14} G , the periods ranging from 5 to 12s, and ages up to 10^4 yr , which is identified as the dominant component of the magnetar ensemble.

In this model it was proposed that high-field neutron stars with ($\log B_s > 13.5$) originate from higher-mass progenitors and a mean mass of NS with high-field is about of $1.6 M_{\odot}$. This value is significantly higher than the mean mass of $1.4 M_{\odot}$ for the ensemble of radio pulsars. The predicted by Ferrario & Wickramasinghe (2006) distribution of surface magnetic fields B_s of massive OB stars is shown in Fig. 5.

We see that this distribution has nothing to do with the real one obtained from the measured magnetic fields of OB stars, which is also plotted in the Fig. 5. It means that the main assumption of the model about the magnetic flux conservation of massive stars is probably wrong.

Recently, Popov et al. (2010) have constructed the population synthesis model of different types of neutron stars taking into account the magnetic field decay and using the results from the neutron star cooling theory. They established that their theoretical model is consistent with the observational data if the initial magnetic field distribution function follows a log-normal law with a mean magnetic field $\overline{\log(B_0)} = 13.25 \text{ G}$, and $\sigma_{\log(B_0)} = 0.6$. In the model about 10 % of neutron stars are born as

magnetars. The magnetic field decays significantly during the first million years of the neutron star lifetime (about a factor of 2 for low-field NSs, and more than an order of magnitude for magnetars).

The mean value of the magnetic field for an ensemble of galactic neutron stars can be found from the relation (9). Supposing that the radius of neutron stars is $R=10$ km, we can find the mean magnetic field for galactic NS: $\overline{\log(B_s)} = 11.96$. This value is lower than the optimal value in the model by Popov et al. (2010) by more than an order of magnitude. Such a large discrepancy can be in principle explained if we take into account the evolution of the magnetic field of neutron stars with time (Pons et al., 2009).

5 Conclusion

We report the results of study of magnetic fields of main sequence stars, white dwarfs and neutron stars. The following conclusions can be drawn:

- The mean magnetic fluxes for massive OB stars are much larger than those for all neutron stars, except magnetars;
- The statistical properties of magnetic fluxes of white dwarfs are consistent with the assumption of stellar magnetic flux conservation, whereas for neutron stars this assumption fails;
- Most of massive stars lose the lion’s share of their magnetic fluxes before they become neutron stars;
- Our magnetic field and magnetic flux distributions can be consistent with the model of isolated neutron stars in the population synthesis obtained in Popov et al., 2010 only if we take into account the magnetic field fading in neutron stars with time;
- The distribution of the magnetic fields of massive OB stars contradicts with the magnetic field distribution of the main sequence massive stars ($8-45 M_\odot$) predicted by Ferrario & Wickramasinghe (2006).

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