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MASSES OF MACROSCOPIC QUARK CONFIGURATIONS IN METRIC AND DYNAMIC THEORIES OF GRAVITATION

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Abstract. Within the bounds of the general relativity and in gravidynamics, spherically-symmetric configurations are considered with the limit equation of state $(P = (\varepsilon - 4B)/3)$ and with the density increasing to the center. It is shown that unlike GR, where the existence of strange stars only is permissible *(u-, d-,* í-quarks), in the consistent dynamic theory of gravitation the existence of *stable* configuration with $\varepsilon \sim r^{-2}$ (quark star) is possible with a 'bag' out of quark-gluon plasma which includes all possible quark flavors *(u, d, s, c, b, t, .. .)*. The total mass of such a compact object with the bag of the radius of ≈ 10 km (whose surface consists of the strange self-bound matter) must be ≈ 6-7 $M_\odot$.

1. Introduction

By the metric theory we mean here, first of all, general relativity (GR) and all versions of gravitational theories which proceed from Einstein's principle of equivalence. A project of theoretical model of gravitational interaction based on the *consistent* application of dynamic principles (gravidynamics) is presented in previous papers (Sokolov, 1990, 1991, 1992a, b, see also the references therein). In gravidynamics (GD) *the law of equivalence* of inertial and gravitational masses is certainly true, but here 'the principle of equivalence' is not used in any way which we consider, following Fock (1961), only a kinematical consequence of the fundamental law: $m_i = m_j$. Accordingly, in the consistent dynamic theory of gravitation the field is not reduced to the space-time metrics which, as in Maxwell's electrodynamics, can *always* be described by Minkovsky's metric tensor.

In the suggested report we try to answer the question why in GR only quark configurations, consisting of strange matter *(u-, d-, s-*quarks) - 'strange stars' - are permitted, while in GD quark-gluon plasma of analogous objects may consist of quarks of all possible types - 'quark stars' proper. As will be seen from the following, the peculiarities of GR and GD become most essential when we consider the utmost in-homogeneous quark configurations in both theories.

There are already many calculations of quark configurations (strange stars) within the bounds of GR (i.e., the calculations on the basis of Oppenheimer-Volkoff's (OV) hydrostatics equations; cf. Haensel *et al.,* 1986; Alcock *et al.,* 1986; Benvenuto and Horvath, 1989; Krivoruchenko, 1987; Øvergård and Østgaard, 1991). The same calculations were carried out recently in the SAO of Russian AS. But unlike other groups, we were interested first of all in the way the modern (dynamic) theory of strong interactions - quantum chromodynamics (QCD) - 'works' in the conditions of the *strongest* gravitational field of a compact object with a mass of $\geq M_{\odot}$. Up to now the corresponding observational information still does not exclude alternatives to GR. Ultimately, our basic purpose is a *test* of the gravitational interaction theory, elucidation of observational consequences and obtaining estimates allowing to compare GR and GD in a description of the same object - compact quark configuration.

2. Utmost Inhomogeneous Quark Configurations in GR

This section addresses the calculations of purely quark configurations described by the *limiting* equation of state

$$P_Q = 1/3\ (\varepsilon - AB), \qquad (1)$$

where ε is the total energy density inside a huge quark bag, $4B/c^2 = \rho_{\mathrm{QGP}}$ is a macroscopic density on the surface $(P_Q = 0)$ of the bag consisting of the quark-gluon plasma (QGP). Equation (1) is the limit to which tends the corresponding total equation of state (Alcock *et al.,* 1986; Haensel *et al,* 1986),describing the media consisting of quarks with masses tending to zero. Quark-gluon interactions stay in the lowest order in a_c (i.e., $2\alpha_c/\pi$ should be sufficiently small so that it remains the first term of expansion in the expression for thermodynamic potential). Below, we speak about sufficiently cold (catalyzed) quark matter at temperatures not more than (for example) 10^{10} K when this matter is already a degenerate Fermi fluid. Thus in this paper the question is on a totally cooling down quark star when electrons are absent in the picture, and with no electrons, there is no corresponding neutrino flux. As a matter of fact, subject to the above remarks, we shall use here an asymptotic MIT bag model (Øvergård and Østgaard, 1991), where the bag constant B is a measure of confinement strength.

Integration of equations of hydrostatic equilibrium (OV equations) gives, in particular, the relation between the mass and the radius of the compact object (Figure 1). Here the value B is chosen as B = 67 MeV fm^{-3}. This corresponds to the macroscopic density on the surface $(P_Q = 0)$ of QGP-bag equal to $\rho_{\mathrm{QGP}} \approx 1.7\ \rho_{nucl}$ ($\rho_{nucl} = 2.8\mathrm{x}10^{14}$ g cm^{-3}).

Usually all calculations of this kind (see, for example, Haensel *et al,* 1986; or Alcock *et al,* 1986) are interrupted near point *C* in Figure 1. This corresponds to the OV limit for a compact object with the equation of state (1). All configurations lying to the left of point *C* turn out to be unstable with respect to small radial perturbations. Branch *C-D* in Figure 1 is closest to the black holes - 'permanently' unstable objects. Such an instability is described in many manuals on GR (Zel'dovich and Novikov, 1971; Shapiro and Teukolsky, 1983).

In Figure 1 the dotted line shows the *M/R* connection for neutron stars. This connection is close to that given by Bethe-Johnson's equation of state. The same connection *M/R* at B = 67 MeVfm^{-3} corresponds at first (i.e., for small masses ≤ 0.5 $M_{\odot}$) to neutron stars, and then to neutron stars with a growing (with further mass increase) quark nucleus, arising inside the compact object when its central density exceeds the value $4B/c^2$ (see, in detail, Haensel *et al,* 1986).

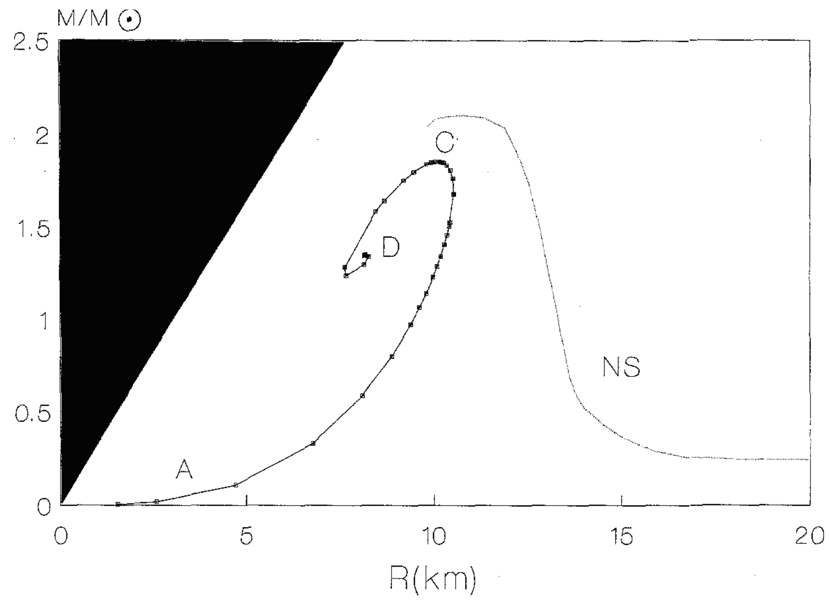

Fig. 1. The mass-radius relation for purely quark configurations with equation of state (1). The dotted line shows the *M/R* relation for neutron stars. Point C correspods to the OV-limit, point *D* indicates the positon of the utmost inhomogeneous quark configuration in GR. The Black Hole region is shaded (see the text), B = 67MeVfm^{-3}.

From Figure 1, it is seen that at the same mass, neutron stars are more extended, or less compact objects, than purely quark configurations, are calculated for limit equation of state (1). In GR only black holes with infinite gravitational red shift z can be more compact objects. Thus, if Equation (1) is really the limit of the equation of quark matter state at superhigh densities ($>\rho_{nucl}$), then it means that, in the bounds of GR, simply, there are no more compact hydrostatic equilibrium configurations than those shown in Figure 1. In this sense, purely quark configurations corresponding to the curve *AC* in Figure 1 are the limit objects for GR. Here we can speak of hydrostatic equilibrium stable compact objects with the surface (z is a finite value) instead of the event horizon.

But the basic difference between purely quark configurations and neutron stars, which is important for us to underline here, is the fact (and it is seen in Figure 1) that the quark matter or strange matter in GR is *self-connected.* Such an object is really a huge quark bag whose hydrostatics at small masses *(A* branch in Figure 1) is guaranteed only by strong (color) interaction. At $M \geq M\Theta$ and at masses close to the OV limit the curve in Figure 1 'turns' to black holes due to GR effects.

Hydrostatic calculations with the use of OV equations give the following dependence on the value of *B* for the maximum mass of purely quark configurations

$$M_{\max}(C) \approx 1.85 \left(\frac{67 \mathrm{MeV} fm^{-3}}{B} \right)^{1/2} M \odot \qquad (2)$$

The analogous formula for the OV limit was obtained by Haensel *et al.* (1986) as a result of numerical calculations including the equation of state (1). From this paper and also from the calculation by Alcock *et al.* (1986) it follows that the basic parameter which determines the value of masses and radii of such dense ($> \rho_{nucl}$) and compact objects - quark configurations - is the value *B*. Our calculations have confirmed the conclusion that the crucial factor is the choice of the macroscopic density value $\rho_{QGP} > \rho_{nucl}$ at the surface of the QGP-bag.

In other words, equation of state (1), in which quarks are considered in the limit as almost free noninteracting massless particles at the calculation of the hydrostatics of objects with such high densities, gives approximately the same values of the 'observed' parameters as the equation of state does, allowing for a finite value of the constant of colour interaction and nonzero mass of s-quark. As it will be seen from the following, this circumstance can be directly *interpreted* in the bounds of consequences of QCD and macroscopic properties of QGP.

The choice of $4B/c^2$ density on the surface ($P_Q = 0$) of the macroscopic quark bag can be determined from the following reasons.

The *upper* value of the constant *B* is determined by the condition of the *self-connection* of quark (strange) matter at zero pressure *P,* formulated by Witten (1984). This condition demands that the corresponding energy per baryon at $P = 0$ should be less than energy per baryon for the most stable non-strange matter - crystal iron: i.e.,

$$E_Q(P=0)=\frac{4B}{n_s}=860.6\text{MeV}\left(\frac{B}{67\text{MeV}fm^{-3}}\right)^{1/4}<930.4\text{MeV}(^{56}\text{Fe}) \qquad (3)$$

Hence, for the B constant, we obtain that $B \leq 91.5$ MeV fm^{-3}, or for macroscopic density of plasma on the surface of the QGP-bag, we obtain $\rho_{QGP} \leq 2.3\ \rho_{nucl}$. It means that at a big leap of density on the bag surface some hadron configuration would be energetically more preferable than QGP.

The *lower* value of B is determined by the fact that with decreasing dimensions and mass of the bag *(A* branch in Figure 1) we shall come ultimately to a model of bags (MIT) or to a 'macroscopic' configuration with baryon number $A \approx 100$ (consisting mostly of *u*- and *d*-quarks?). Thus, the masses of macroscopic quark configurations must be connected with hadrons mass spectra by the (semi-empiric) relation (Chodos *et al.,* 1974)

$$B \geq B_{\text{MIT}} = 0.13\,|\,\varepsilon_V| \approx 67\text{MeVfm}^{-3}, \qquad (4)$$

where $|\,\varepsilon_V|$ is the energy density of QCD-vacuum equal to ≈ 0.5 GeV fm^{-3} (Novikov *et al.,* 1981).

Thus the density at the surface of the macroscopic quark bag is somewhere within the limits

$$2.3\rho_{\text{nucl}} \geq \rho_{\text{QGP}}(\text{P}_\text{Q} = 0) \geq 1.7\rho_{\text{nuol}}. \qquad (5)$$

Accordingly, maximum mass of purely quark configuration must be in the limits 1.58 $M_\odot \leq M_{max}(C) \leq 1.85\ M_\odot$. This is another consequence of the calculations of the kind (Haensel *etal,* 1986; Øvergård and Østgaard, 1991): namely, the application of the modern phenomenology of strong interactions reduces considerably the value of the OV-limit. In the old phenomenology with Yukawa's potential and the exchange of vector mesons, where the equation of the type of $P = \varepsilon$ is used as a limit equation of state at $\varepsilon/c^2 \geq \rho_{nucl}$, the corresponding value of the OV-limit is more than 3 $M_\odot$ (Rhoad-es and Rumni, 1974; Shapiro and Teukolsky, 1983).

To understand why, in GR, *only strange* stars are possible it is necessary to apply to the profiles of (energy) density the corresponding hydrostatically balanced quark configurations. In Figure 2 the behaviour of (energy) density is shown inside the quark bag as dependent on the distance from the center of a spherically-symmetric configuration. This density profile corresponds on the OV-limit - i.e., this is $\varepsilon(r)/c^2$ for the last stable hydrostatically balanced configuration which can exist in nature, if the equation of state (1) for QGP is true and, of course, if GR is true. Thus, GR, together with the limit equation of state (1), imposes the limitation on the maximum achievable density QGP (density in the center):

$$\rho \leq \rho_{GR} \approx 10\,\rho_{nucl} \approx\ 28 \times 10^{14}\ \mathrm{g\ cm^{-3}} \tag{6}$$

and, consequently, on numerical value of the OV-limit.

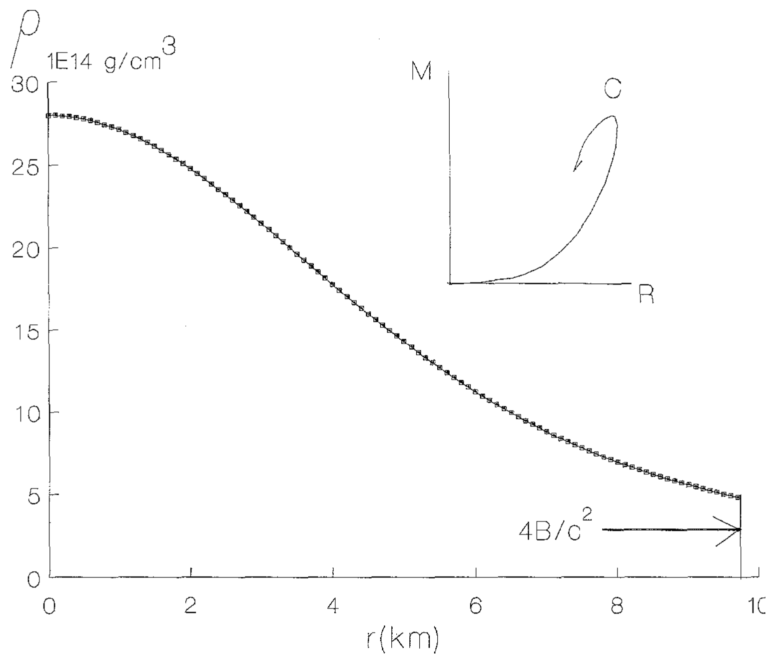

Fig. 2. (Energy) density profile for the last stable (OV-limit) spherically-symmetric configuration corresponding to point C in Figure 1. The distance r from the center of QGP-bag is measured in km, total

density in g cm^{-3}, B = 67 MeV fm^{-3}.

All other stable configurations on the basis of Equation (1), lying to the right of point C, are even more homogeneous. In the limit, as was mentioned above, at $r \rightarrow 0$ *(branch A* in Figure 1) we deal ultimately with the model of MIT-bag with an absolutely homogeneous profile $\varepsilon(r)$ at baryon number $A \approx 100$. We emphasize here once more that purely quark configurations, corresponding to points on the curve from A to C in Figure 1, are the most homogeneous ones from possible stable compact configurations corresponding to OV-equations (Alcock *et al.,* 1986).

After all, we can calculate the profile $\varepsilon(r)/c^2$ of the *utmost inhomogeneous* hydrostatically balanced configuration corresponding to point D in Figure 1. This profile is shown in Figure 3. The density in the center of such an object tends to infinity and falls, with r increase, very close to a law

$$\varepsilon(r) \sim r^{-2}.$$

But such configurations, according to GR, are never realized in nature since they are mostly unstable with respect to small radial perturbations (Zel'dovich and Novikov, 1971) and during the time of the order of *R/c* they must collapse into black holes. But nevertheless we consider the situation in more detail, since it is such a configuration (forbidden in GR with a big 'margin') which can be realized as a stable stationary state in the bounds of the dynamic alternative to GR - in gravidynamics.

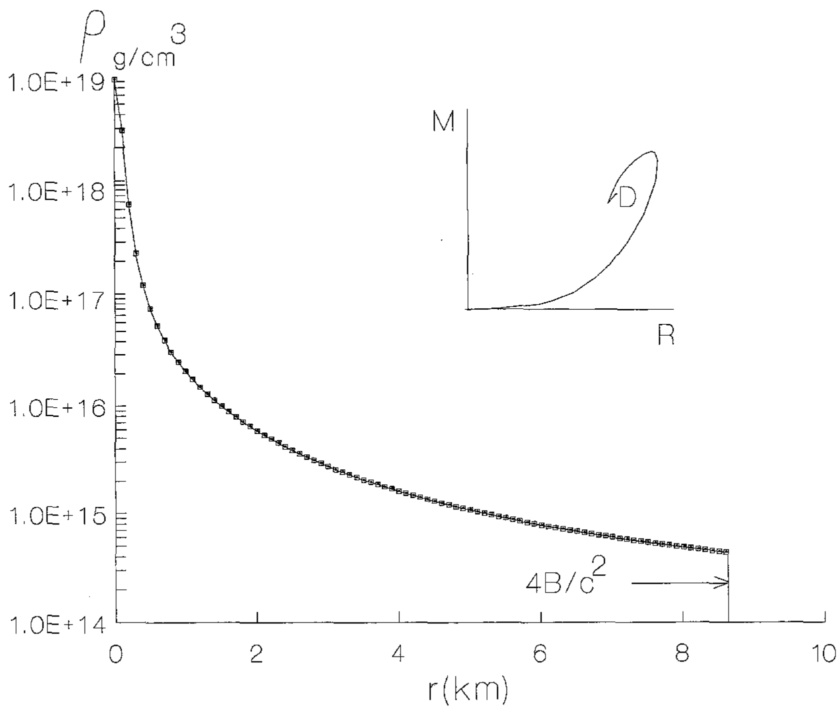

Fig. 3. (Energy) density profile for the utmost inhomogeneous hydrostatically-balanced configuration with equation of QGP state (1) calculated in the bounds of GR. The density on the QGP-bag surface is taken to be equal to $\rho_{QGP} = 1.7\ \rho_{nucl}$ $(B = 67$ MeV fm$^{-3})$.

For macroscopic density at the bag surface, in accordance with conditions (5), we choose for definiteness some average value

$$\rho_{QGP}(P_Q = 0) = 2\rho_{nucl} \approx 5.6 \times 10^{14}\ \text{g cm}^{-3}. \tag{7}$$

In accordance with QCD (the theory of colour interactions) and also in accordance with what is known about expected properties of QGP (Emel'yanov *et al.,* 1990; Collins and Perry, 1975), it can be considered that the bag surface consists mainly of the lightest *u*- and *d*-quarks which come first in the state of deconfinement *at such* a gigantic macroscopic density. It can be interpreted (analogously to usual plasma) that the free path length l (relative to color interactions) of *u*- and *d*-quarks in such a plasma with $\rho \geq \rho_{QGP}$ becomes either equal or even much greater than $l_c \approx 1$ fm; l_c being the characteristic radius of strong interaction or the radius of confinement. In the end, l can simply become a macroscopic value comparable with the bag dimension. These are just the quarks for which at $\rho \approx \rho_{QGP}$ the equation of gas state of asymptotically free quarks (1) turns out to be true, since for them we can assign $m_u \approx m_d \rightarrow 0$

(at such characteristic transmissions of momentum in QGP which correspond to particle interactions in such dense matter).

A heavier s-quark $(m_s \approx 200$ MeV) at $\rho \geq \rho_{\text{QGP}}$ exists in plasma as heavy 'admixture' as a result of equilibrium reactions of the type

$$u + d \leftrightarrow u + s$$

(see the review by Haensel, 1987). Color interaction of this ('more non-relativistic', than u and $d)$ quark is still rather strong ($\alpha_c > 0.45$) at $\rho \approx \rho_{\text{QGP}}$ and that is why the s-quarks must have smaller mean-free-path in QGP than 'massless' u- and d-quarks.

So one can say that a heavier s-quark arising at weak interactions at $\rho \approx \rho_{\text{QGP}}$ is still mainly in the volume of the confinement $l^3_c \approx 1$ fm^3. Density boundary over which the deconfinement or almost total 'defreezing' of s-quark occurs is rather close to ρ_{QGP}. The 'defreezing' of j-quark occurs when the macroscopic density becomes greater than

$$\rho \equiv \rho_{\text{QGP}} + m_s c^2/l^3_c \approx 9.1 \text{ x } 10^{14} \text{ g cm}^{-3}, \tag{8}$$

in accordance with the interpretation of chemical potential, as the change of energy density at the unit change of concentration of particles of a given kind. In other words, at $\rho > \rho_s,$ in every 'cell' of ≈ 1 fm^3 volume, there is already more than one s-quark and it becomes just as 'tight' for them as it was at $\rho \geq \rho_{\text{QGP}}$ for u- and d-quarks. Yhen the colour interaction of s-quarks must become weaker (α_c <0.45). Accordingly, at QGP densities greater than $\rho_s,$ s-quarks also can be considered relativistic $(m_s \rightarrow 0)$.

As a matter of fact, the confirmation of such a logic of'defreezing' of s- (and heavier) quarks is the fact noted in the quoted paper by Haensel *et al.* (1986) and mentioned before: namely, that the determining parameter of the equation of state for QGP is the value B or the density value at which QGP is formed. The direct calculation of the hydrostatically balanced configuration with the limit equation of state (1), not only reproduces the results of that paper, where the authors, besides the $B,$ allow also for $\alpha_c = 0.45$ and $m_s c^2 = 200$ MeV, but varying $4B/c^2$ value in (1), one can reproduce, with an acceptable precision, almost all the results of calculations by Benvenuto and Horvath (1989) with other parameters α_c and m_s. Hence, we conclude that indeed the densities ρ_{QGP} and ρ_s (at which í-quark can also be considered a free massless particle) must be close $\rho_{\text{QGP}} \sim \rho_s$. I.e., Equation (1) can actually be applied without paying attention to the essential difference in masses between u-, d-, and s-quarks.

Or, in other words, the following *interpretation* of the results of all the mentioned calculations is possible. At $\rho_{\text{QGP}} \leq \rho \leq \rho_s$, the heavier quark is present in QGP as a 'heavy admixture' and does not distort strongly the limit equation (1). At $\rho > \rho_s$ heavy quarks, as well as lighter ones, are also in the state of deconfinement and so here Equation (1) becomes applicable again.

From what is said above, it becomes clear why in GR only strange stars are possible. As has been noted, the last stable hydrostatically-balanced configuration (see Figure 2) imposes limitation on density (6). Only then can one speak of strange stars, following the same logic, the deconfinement ('defreezing') of even heavier c-quark *(*$m_c c^2$ =1.4 GeV) must occur at densities greater than the limit density $\rho_{\rm GR}$. In every 'cell' of l^3_c volume there can be at least one *c*-quark if the macroscopic density turns greater than

$$\rho_c \equiv \rho_s + m_s c^2/l^3_c \approx 33.9 \text{ x } 10^{14} \text{g cm}^{-3}. \quad (9)$$

In that case the color interaction becomes so weak that the corresponding α_c becomes even less. Therefore, one can consider for c-quark, that $m_c \rightarrow 0$. Of course, for the configuration in Figure 2, which is still attainable in GR, some admixture of c-quarks can appear in plasma near the center as a result of some $(d + u \rightarrow d + c)$ weak processes analogously with the admixture of s-quarks at $\rho < \rho_s$.

Returning to the utmost inhomogeneous configuration with the density profile in Figure 3, at $\rho > \rho_s$ in plasma there must be a lot of relativistic charmed quarks $(m_c \rightarrow 0)$ and like the case of s-quark the properties of QGP are described ultimately by limit equation (1). But *all* the configurations with the density in the center greater than p_c, as well as the utmost inhomogeneous configuration, are to the left of the OV limit in *M/R* curve (point C in Figure 1). Hence, if GR remains true even in such a strong gravitational field then in nature there exist only strange stars as maximum compact stationary objects with the surface (i.e., with a finite *z)* but not with the event horizon. ('Charmed stars' do not exist in GR.) The utmost inhomogeneous configuration (point *D* in Figure 1) in which *all* quark generations would be 'defrozen' is not realized either, according to GR – it is 'eaten up' by black holes.

By reasoning of this section, we tried to make more concrete analogous speculations expressed by Alcock *et al.* (1986), in connection with their use of the same equation of state, independent of the number of particle flavors. Since if instead of the full expressions we use their limit (1), then, strictly speaking, everything that is said about quark masses, quark flavors, chemical potentials, number density of different flavors of quarks, density (ρ) at which 'the appearance' of the next flavor occurs and even the use of some finite value α_c , is now only *interpretation* in the bounds of perturbation QCD. The results of calculation of hydrostatically equilibrium configurations are determined in the end only by the value $4B/c^2$ or the energy density at which the limit (1) can be used.

Certainly, one should try to understand why the results of such calculations differ in less than 4% from the results of calculations with the help of full expressions. That is why where we use the notion of 'defreezing' of the next flavor, meaning first of all the fact that for this 'new' flavor, at the given density, the condition $l > l_c$ begins to be fulfilled.

Besides, in so-called 'full expressions' the used values of quark masses and the value of α_c are fixed. At the same time it is known that quark masses are measured at definite effective transferred momentum Q. Numerical values which are usually used concern the distances between the quarks of the order of 10^{-14} cm, that in our case means a definite *macroscopic* density ρ. The less the distance between interacting quarks, i.e., the greater macroscopic density, the less quark masses can be. But this simply means that there must exist a dependence of α_c on ρ (more details will be in the next section). In other words,

$$m_q(\alpha_c) = m_q[\alpha_c(\rho)] = m_q(\rho);$$

and then at calculations of hydrostatically equilibrium configurations only macroscopic density $\rho \geq 4B/c^2$ becomes really the basic parameter.

Then even in the case of the limit (1) it is not absolutely necessary to require that, for all ρ's, te fulfillment of the equality $\alpha_c = 0$. For *u*- and *d*-quarks defreezed at $\rho > 4B/c^2$ at a given p, we can put $m_{u,d}(\rho) \rightarrow 0$, then $\alpha_c(\rho) \neq 0$. To have a possibility to use the perturbation theory the value $\alpha_c(2\alpha_c/\pi < 1)$ must be sufficiently small. At densities $\rho < \rho_s$ (8), the heavier quark s is present in plasma, but for it l is still rather small $(l \leq l_c)$ for its contribution into pressure could change the equation of state essentially. At densities $\rho > \rho_s$ in the full equations besides decrease of $\alpha_c(\rho)$, the value m_s may also tend to zero, that leads in turn to the disappearance of corresponding ('massive') terms in the full expressions. In the end we shall be even closer to the limit (1).

All this reasoning could be illustrated by corresponding calculations, but the matter is that nobody know today the precise form of the dependence of $\alpha_c(\rho)$ and $m_q(\rho)$ and we can only guess (see the next section) how α_c will behave at high and superhigh ($\rho >> \rho_{nucl}$) macroscopic densities. However, it may be the essence of differences between quark configuration calculation results in the bounds of asymptotic MIT bag model, i.e., with the help of Equation (1), and ones in the bounds of the Perturbative QCD model (Øvergård and Østgaard, 1991).

In conclusion of this section which has been dedicated to the pure quark configurations in GR. We emphasize once more that the compact object consisting of strange matter with the equation of state very close to (1) *is the last* opportunity of stable state after which only black holes with z →∞ follow.

3. Quark-Gluon Plasma in Gravidynamics

'Quark stars', considered here as objects consisting of QGP including all possible flavors *(u, d, s, c, b, t,...)* of quarks, turn out to be unstable in GR. One can say that such objects must not exist in nature according to all versions of GR as well in which there are 'frozen stars'.

A 'quark star' as a *limiting* (in several senses) stable object with the total mass M_Q with the QGP-bag surface ($z \neq \infty$) of the radius of $R_{QOP} \equiv GM_Q/c^2$ (inside the Schwartzschild sphere according to GR) can exist if we adhere to the dynamic, totally non-metric description of gravitational interaction.

In gravidynamics - GD (unlike geometrodynamics: GR) - the profile of the total (energy) density $\varepsilon(r)/c^2$ of analogous quark configuration does not terminate in a vacuum (see Figure 4). Around the macroscopic quark bag (QGP-bag) a fur-coat exists - the 'gas' of virtual gravitons whose energy density has to be allowed for in the equation of state.

Here the *total mass* M_Q of the object entering the determination of $R_{QGP} \equiv GM_Q/c^2$ should be found in 'long-wave limit' (like classic charge of electron); i.e., at $r >> R_{QGP}$ or, in other words, the mass is determined by Newtonian rules. If

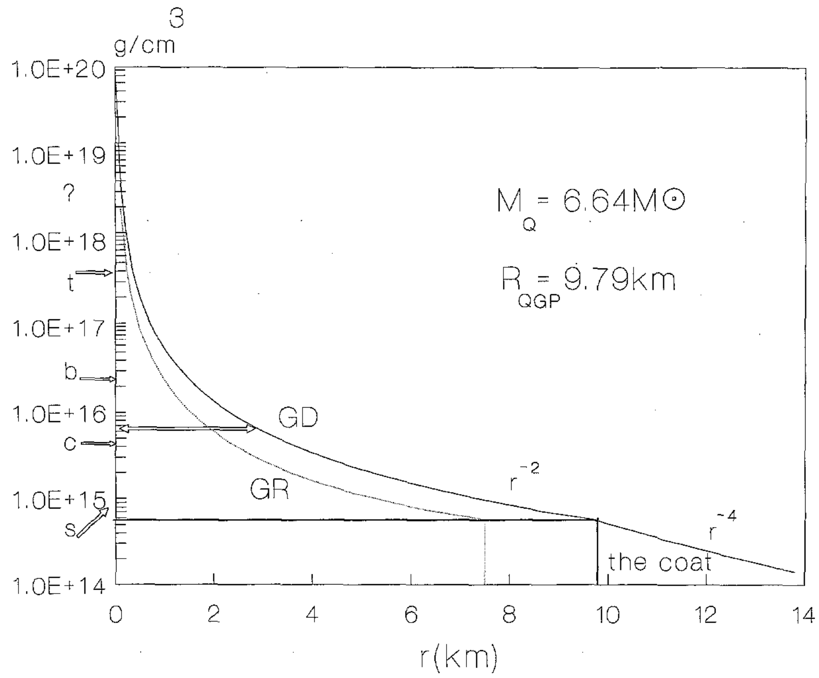

Fig. 4. Density profile for a quark star in GD (solid line). Fat rectangle shows 'the background' created by gluons (see the text) distributed homogeneously (?) in the bag with $R_{QGP} \approx$ *10 km. 'Vacuum' around the bag is filled by a 'gas' of virtual gravitons (the fur-coat) with energy density* $\theta^{00}(r)$. The densities are indicates at which 'the defreezing' of *s, c,b,t, ...* quarks occurs. The arrow indicates the density at which the perturbative QCD vacuum must be totally restored ($\alpha_{cm} \approx 0.7$). The dotted line represents the density profile of analogous quark configuration in GR for *4B:* $c^2 = 2\rho_{\text{nucl}}$ (B = 78.5 MeV fm^{-3}).

the object is described by 4-potential (4) from the paper by Sokolov (1992b) with the radius of the bag $R = R_{QGP}$, then M_Q consists *by half on* the 'coat' mass. Thus for the total energy M_Qc^2 of the quark star in GD one can write

$$\tfrac{1}{2} M_Qc^2 \text{ (the bag with } R = R_{QGP}) + \tfrac{1}{2} M_Qc^2 \text{ (the 'coat' in vacuum)} = M_Qc^2 .$$

Thus, the mass of the whole configuration is determined by integration from its center ($r = 0$) and up to $r = \infty$o. (This is an essential difference from the definition of mass in GR.) Strictly speaking, this is the determination of mass of any objects in GD. By force of this fact in GD there is no event horizon that is ultimately the result of total refusal of geometric phenomenology constituting the base of GR. The bag surface can undergo some unselected sphere r = $2GM_Q/c^2$ as a result of the relativistic collapse, but the *whole* gravitating object, including its gravitational 'atmosphere' (or coat), can *never* be found under this 'horizon'.

For the object, with limit achievable parameters, which a cold quark star if $R_{QGP} = 2GM_Q/c^2 \approx 10$ km seems to be in GD, the equation of state inside and outside the bag is the same: i.e.,

$$P = 1/3\ \varepsilon. \tag{10}$$

Inside the bag the total pressure is a sum of two ('partial') pressures. First of all it is the pressure

$$P_Q = 1/3\ (\varepsilon - 4B).$$

Strictly speaking, this formula gives the pressure of degenerate Fermi-gas of free and massless quarks. Then the contribution of gluons (with some admixture of gravitons?) in the limit under investigation is determined by the equation

$$P_G = 1/3\ (4B). \tag{11}$$

Hereafter, we shall proceed from the assumption which is apparently true in the case of huge ($\rho >> \rho_{nucl}$) macroscopic densities in question. We consider that gluons which are almost free and almost non-interacting with each other (at the total density increasing to the center) can be distributed inside the bag with constant and positive density $4B$. The interaction of gluons becomes essential far from the center, maybe even near the very 'wall' of the macroscopic bag, where (as a result of that) the energy-momentum tensor trace of massless gluon field becomes non-zero... We do not know so far at what distance from the bag wall it will occur, that is why we choose here 'the simplest' limit case (11).

As a result of it, in the sphere $r = R_{QGP}$ the sum of pressures is equal to (10). The *total* energy density in the bag with $R_{QGP} \approx 10$ km decreases from the center according to the equation

$$\varepsilon(r) = \frac{4B}{c^2} R_{QGP}^2 r^{-2} \tag{12}$$

(Sokolov, 1991, 1992b), if the equation of state inside the bag is taken in form (1). The QGP-bag in GD turns out to be connected only by colour forces ('wall' of bag), the gravitation inside the bag is 'switched off for such a *limiting* object which the quark star is. It can be assumed that in GD, in that case, QGP is in a totally (at $R = R_{QGP}$) stress-free, self-bound state when there are no forces binding the bag besides colour ones. Then the distribution *e(r)* is here maximum inhomogeneous. It differs radically from an analogous case in GR at totally homogeneous distribution of density (dε/dr→0) when M→0 also.

Outside the bag (in 'vacuum') 'a gas' still remains from virtual gravitons with the same equation of state (10). *Positive* energy density $\varepsilon \equiv \theta^{00}(\tau)$ of the gravitational field falls here from the value 45 (at the boundary of the two 'mediums': QGP - 'vacuum') according to the law

$$\theta^{00}(r) = \frac{4B}{c^2} R_{QGP}^4 r^{-4} \quad , \tag{13}$$

which is connected with the fact that at distances $r \geq GM_Q/c^2$ the self-action of gravitons arises (scalar and tensor gravitons, tensor and tensor ones; see Sokolov, 1990, 1991, 1992b).

Of course, it is not excluded that in QCD-theory which would be more correct than the bags model, the contribution of quarks and gluons in the total energy density (12) inside the QGP-bag could be distributed in an absolutely different manner than we assume here. But (most probable) these are such contributions of fermion and boson components at the huge ($\varepsilon/c^2 >> \rho_{nucl}$) density which increases towards the center ($\varepsilon \sim r^{-2}$), that as a result only such limit equation of state (10) conforming to the rest of physics turn out to be true.

Strictly speaking, from the very beginning, the question in (1) was only on quarks since here there is no explicit contribution into pressure which corresponds to gluons. To make sure of that it is sufficient to look at the full equation of state (Alcock *et al.,* 1986) before the pass to the limit (1). Thus, in the case of quark configurations in GR one cannot say about QGP: in such a plasma there are simply no gluons, if using the limit equation (1) as the equation of state. In GD we do deal with QGP since in GD we try to account *explicitly* for the contribution of bosons, at least in the case of asymptotically free gluons with the equation of state (11). The last can be justified apparently only in the case of macroscopic density of QGP (12) increasing to the center, when the 'constant' of color interaction decreases sufficiently quickly with the increase of ε from the wall of the bag towards its center. Below, in this section, it will be said that it is possible, in principle, at such a profile of *ε(r)* as (12).

To obtain the *total* (observable) mass of such a quark configuration (unlike what was in GR) at the integration of ε/c^2 it is necessary to allow for the energy θ^{00} of the gravitational field itself. As a result, the mass of a quark star in GD can be expressed in terms of the energy density value at the boundary between QGP and 'vacuum' as

$$M_Q = 6.64 M\odot \left(\frac{2\rho_{nucl}}{4B/c^2} \right)^{1/2} ; \tag{14}$$

and the same restrictions (5), which were mentioned above, fix the mass and the radius of the cold quark star in GD in the limits

$$6.21\ M_\odot \le M_Q \le 7.25\ M_\odot , \qquad 9.16 \text{ km} \le R_{QGP} \le 10.69 \text{ km} . \tag{15}$$

The lowest value of the total mass of the configuration and, accordingly, the lowest value of the QGP bag radius (as was said above) follow from the condition that at the density (12) on the bag surface equals $\varepsilon/c^2 \approx 2.3\ \rho_{\text{nucl}}$ (3,5). This surface consists from strange self-connected (i.e., stable at $P_Q = 0$) matter. Since ε increases towards the center of the bag and, consequently, all other kinds of quarks become defrozen, then in GD (unlike what was in GR) it is necessary to speak not about a strange star, but about a cold quark star with the strange surface, if the density on this surface does not exceed $\approx 2.3 p_{\text{nucl}}$.

If we assume that some other conditions of the type of (3) are possible but at $P_Q \neq 0$, when at bigger densities on the QGP surface, already more massive quarks than s-quark become 'defrozen', then according to (14) the masses of corresponding meta-stable quark configurations will be less than $6 M_\odot$ down to the values $M_\odot \to 1.4\ M_\odot$. But in any case, Witten's condition (3) at $P_Q = 0$ fixes some maximum mass (> 6 $M_\odot$) of the *most stable limit* quark configuration in GD. Whether such a limit object can exist allowing for astrophysical reasons is another question. But such a limit can be a consequence in principle of GD and QCD, if GD gives a more or less correct description of the strong gravitation.

Thus, as follows from a brief review of properties of compact objects, collapsars (Sokolov, 1991) - from masses and radii of such objects in binary systems such as Cyg X-l, A0620-00, LMC X-l, LMC X-3 - can readily be considered as 'candidates' into quark stars of GD with the strange surface. Some properties of the quark star which could lead to corresponding observational manifestations are discussed in more detail in the quoted review. But apparently, the basic *observational consequence* confirming the version of GD u QCD suggested here could be indeed the existence of a selected mass value of collapsars (or candidate into black holes) ≈ 6 - 7 $M_\odot$ (15). Since in GR there is not preferable mass values of black holes for all masses of 'candidates' greater than OV-limit, then one will have to invent some astrophysical (e.g., evolutionary) arguments explaining the mass of a 'typical' collapsar in these close binary systems.

In what follows, we shall try to imagine how the 'running' constant α_c of the strong interaction could depend on the parameters of the QGP bag under consideration. And, most importantly, what could be the dependence of this value on macroscopic density so that we could make at least the rough agreement between the asymptotic MIT bag model and *the interpretation* of results with the help of the perturbative QCD which we mention here rather often?

In Figure 4 we marked macroscopic densities above which the corresponding quarks are already in the state of deconfinement. The total density *ε(r)* increases deep into the bag and, consequently at the approach to its center the quarks become compressed still tighter. In other words, when the macroscopic density exceeds a certain level, the corresponding quarks are situated relative to each other and *interact with each other* at distances Δl less than $l \approx 10^{-13}$ cm. The same can be said about any particle of QGP. In particular, the known formula for the 'running' constant α_c of color interaction can be written in the form

$$\alpha_c(\Delta l^2) = \frac{12\pi}{(33 - 2n_f)\ln(l_c^2 / \Delta l^2)} \ , \tag{16}$$

where n_f is the number of 'defrozen' quark flavors; l_c, the radius of confinement; and Δl, the distance between two *neighbouring* strongly-interacting colour particles *($\Delta l \neq l$; l being the* free-path length of quarks in QGP which can be much greater than l_c*).*

For example, if one demands that *u*- or *d-quarks* ($n_f = 2$) could be considered almost free ($\alpha_c \approx 0.45$, see Haensel *et al.,* 1986), it is necessary that the mean distances Δl, to which the quarks must be 'compressed' in such QGP would be about $0.25 l_c \sim 10^{-14}$ cm. One may think (see the previous section) that these are just the conditions ($\Delta l < l_c$, $\alpha_c < 1$) that are realized in plasma first for *u*- and *d*-quarks at the *macroscopic* density$\rho \equiv \varepsilon/c^2$, greater than the density at the boundary of the QGP-bag (i.e., greater than the density of phase transition in the QGP state).

If we apply the idea of coupling constants depending on density (which is widely used in 'standard' cosmology of Big Bang for our case of *macroscopic* bag) one can try to parameterize the *macroscopic* constant of strong interaction by the equation

$$\alpha_{cm}(\rho) = \frac{12\pi}{(33 - 2n_f)\ln(\rho / 4B / c^2)} \ . \tag{17}$$

We emphasize here that p is macroscopic and, therefore, a certain *mean* density which does not exclude that microscopic fluctuation of density in volumes of $\sim (\Delta l)^3$ can, generally speaking, exceed p dozens of times. In particular, if we remember here the attempts of getting 'hot' QGP on colliders in volumes of the order of several fm^3, then the corresponding macroscopic density (here the macroscopic volumes of averaging ~ 1 cm^3 are also meant) will be simply zero. In that case there is no question about any gravitational effects which become essential only at big macroscopic masses. Here we mean cold catalyzed self-connected matter which must be the source of the gravitational field.

On the other hand, the strong interaction which is realized here in big (macroscopic) volumes has evidently the character of macroscopic color interaction, something like 'color gravitation' inside a huge self-connected QPG-bag. Formula (17) could describe just such a macroscopic interaction, when the exchange by gluons between the elements of volume inside such a bag, situated at macroscopic distances relative to each other, becomes essential.

Thus here, in our case, it is necessary to speak already about QGP in astrophysical conditions which differ considerably from corresponding conditions available in experiments on the Earth.

Of course, formula (17) can be considered still only as an attempt of some rude extrapolation in the region of superhigh ($\rho >> \rho_{nucl}$) densities. In particular, the calculation of Δl - mean distance between quarks at a given $\rho>4B/c^2$, the mean free-path 1 and also other parameters of QGP will demand further study of microscopic properties of such plasma as it is made for ordinary plasma. Here we are to meet the same problems that exist in QCD and in quark bag models (in particular). Especially since the notion of the macroscopic QGP-bag can be used *directly* at $\rho \sim r^{-2}$. Really, by use of formula (12), the macroscopic constant α_{cm} of colour interaction (of *macroscopic* volume elements situated at macroscopic distances from each other) inside the QGP-bag can be expressed in terms of r - the distance from the bag center – as

$$\alpha_{cm}(r^2) = \frac{12\pi}{(33-2n_f)\ln(R_{QGP}^2/r^2)} \quad . \qquad (18)$$

Then, the color confinement in the gigantic bag of R_{QGP} radius is provided here in the same way as it was in the model of the *homogeneous* (in density) MIT-bag with the 'radius' l_c in formula (16). In particular, the values α_c and α_{cm} must be in approximately the same relation as the value $|\varepsilon_v|$ - the energy density of QCD-vacuum (obtained from the analysis of sum laws) and the value of the constant B_{MIT} (which is connected with hadron mass spectrum). It naturally follows from the fact that in (17) we actually use directly the model of quark bags.

If, finally, some *definite* value of density at the boundary of the QGP-bag in accordance with the restrictions (5) is chosen, then in evaluation of value a_{cm} one can use the equation

$$\alpha_{cm}(\rho) = \frac{12\pi}{(33-2n_f)\ln(\rho/2\rho_{nucl})} \ . \quad (19)$$

Equations (18) and (19) should be considered here only as an attempt to make the interpretation by means of perturbative QCD, to which we resorted in this and previous sections, agree with the asymptotic MIT bag model. Of course, such equations can be an approximation as the 'initial' approximated equation (16) itself. The most probable, the more precise dependence $a_{cm}(\rho)$, will lead to an even more quick decrease of color forces in the direction from the bag wall towards its center. In the end we shall have to use only limit (1) for the cold, catalyzed QGP. It may be, at this limit, the difference between MIT-bag and QCD approaches will decrease or disappear altogether. We can, therefore, speak already about a *classical limit* of QCD inside the macroscopic bag.

But one way or another, at consideration of macroscopic QGP (i.e., QGP in astro-physical conditions) and the use of QCD, an absolutely definite dependence for $a_c(\rho)$ or $a_{cm}(\rho)$ will be needed for sure. Below we show by corresponding estimations the fact that the approximate formulae (19) suggested above does not contradict considerably to 'the standards' of QCD.

If we consider that the perturbative vacuum is restored at such macroscopic densities ρ_{QCD} that the relation of ρ_{QCD} to the density on the bag border is the same as the relation (4) between $|\varepsilon_v|$ and B_{MIT},

$$\rho \equiv \rho_{QCD} \approx 2\rho_{nucl}\frac{|\varepsilon_V|}{B_{MIT}} \approx \frac{2\rho_{nucl}}{0.13} = 43.07\times 10^{14}\, g\ cm^{-3} \quad (20)$$

(when four kinds of quarks are defrozen, see Figure 4), then Equation (19) yields $\alpha_{cm} \approx 0.74$ at $\rho = \rho_{QCD}$. It is close indeed to the QCD-value $\alpha^{QCD}{}_c$ (1 GeV) = 0.7. At the same time, for strange matter (n_f= 3) at macroscopic densities, when $\rho \approx 15^{15}$ g cm^{-3} < ρ_{QCD}, the macroscopic constant of colour interaction turns out to be equal to $\alpha_{cm} \approx 2.4$. It is close to the value of $\alpha^{MIT}{}_c$ = 2.2 determining the mass splitting of hadron multiplets (Bogolyubov, 1968; De Grand *et al.,* 1975).

Thus both formulae (19), and from estimation (20), it follows that the QCD-vacuum (perturbative vacuum) in strange matter is not restored yet ($\alpha_{cm} \approx 0.87$), which agrees with calculations in the bounds of GR carried out by Kondratyuk *et al.* (1990). As follows from Figure 4 and formulae (19), (20) the perturbative vacuum is totally restored in the interior of a QPG-bag of a quark star in GD at the depth $R_{QGP} - r \approx 7$ km, i.e., in the case of the most stable (limit) quark configuration with the bag whose surface consists of strange matter.

4. Conclusions

From formula (19) it follows, in particular, that in the very center of the QGP-bag with $R_{QGP} \approx 10$ km (i.e., for $r = 10^{-13}$ cm), the macroscopic constant of color forces is only about 3 constants of electromagnetic interaction. The density p here must be about 5.4 x 10^{52} g cm^{-3}, and the mass (in r = 1 fm sphere) must equal 7 x 10^{14} g. Of course, at mutual distances between QGP particles much less than l_c (10^{-17} cm) and, correspondingly, at densities >> 10^{16} g cm^{-3} the constants of all the three fundamental interactions (strong, weak, and electromagnetic) must in the end become indistinguishable from each other.

Thus, in the interiors of a GD quark star - a stationary stable object with density increasing to the center according to the law $\varepsilon(r) \sim r^{-2}$ - just the physical conditions can be realized under which all the interactions unite in one fundamental interaction. For sure, the constants of weak and electromagnetic interactions inside the QGP-bag of the quark star can be also expressed in terms of *macroscopic* densities. And it means that the ideas of *Grand Unification* of all interactions could be tested without resorting to cosmology of Big Bang, but studying the *same* physics of *superhigh densities* (or 'cosmomicrophysics'), observing compact objects (bright X-sources, γ-ray bursts, remnants of supernova explosions, etc.) of stellar masses.

Of course, it should be admitted here that GR describes erroneously the strong gravitational field of such objects. But ultimately, if we abandon the conviction *{a priori)* of *absolute* correctness of GR and do not forget that only a *sufficiently* complete experimental (observational) study of strong gravitational fields can affirm or refute this conviction, then all the preceding discussion can be considered as a possible alternative to black holes of GR.

We recognize that sometimes statements of certain things look schematic here. The matter is that many QGP properties and QGP itself is only a hypothesis, although a hypothesis which follows naturally from the experiments on colliders and from the theory of quarks and leptons. But even now from all what has been said, it is clear that a consistent *direct* allowance for localizable positive (like in the case of all other gauge fields) energy of gravitational field changes completely the collapsar physics. In particular, one of the observational (experimental) arguments in favour of such or similar physics would be the existence of some selected value for the collapsar mass (or for 'the candidates in black holes' of GR). Proceeding from the theoretical scheme developed here, we consider that the collapsar - a compact object with its mass exceeding, certainly, the pulsar mass (or OV-limit in GR) - can be identified with the (limit) cold quark configuration in GD whose mass is 6.2-7.2$M_\odot$.

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