Magnetic stars, 2004, 142-151

A quadrupole model of the magnetic field of βCrB

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Abstract. A model of the magnetic field of the star βCrB is constructed by the method of the "Magnetic Charge Distribution". The magnetic field is described by superposition of two decentered dipoles, which are arranged in the equatorial plane perpendicularly to the axis and oppositely directed to the center of the star. The dipoles form an *irregular quadrupole* and produce on the surface four magnetic spots with a maximal magnetic field strength at the poles $B_p = 14.5$ kG. The angle of declination of the rotation axis to the line of sight has been ascertained by the quadrupole model to $i = 13°$. In consideration of all aspects the idea is advanced of very strong influence of inhomogeneous distribution of chemical elements on the measurable phase curves of the effective magnetic field. Apparently, the star has passed a convective phase in an early stage of evolution.

Key words: stars: chemically peculiar – stars: magnetic fields – stars: individual: βCrB

1 Introduction

The chemically peculiar CP star βCrB (HD 137909, F0p) is one of the most investigated magnetic stars because of the brightness and sharpness of its spectral lines. Stibbs (1950) and Deutsch (1954, 1958) explained the variability of the observed integral magnetic field strength with a period of 18.497 d (Steinitz 1964, 1967) by a model of an obliquely rotating magnetic dipole, which is rigidly bound to the stellar body and might be shifted off the center of the star (Landstreet 1970). Deutsch (1970) was the first to describe the typical spherical asymmetry by moments of dipoles, quadrupoles etc. of spherical harmonics.

An early attempt to explain the phase relation of the effective field B_e by an *equatorially symmetric rotator model* has been made by Oetken (1977). She derived from the shape of the phase curve a prevailing quadrupole moment over the dipole moment by u_{20}/u_{10}=2.0.

Modeling of the magnetic field of βCrB has been also performed by Bagnulo et al. (2000, 2001), who describe the stellar surface field by a *second-order expansion* of spherical harmonics on the base of a centered dipole plus a non-linear quadrupole.

In the present paper the magnetic field structure of βCrB is analyzed by the modeling method of the "*Magnetic Charge Distribution*" (MCD), which is described by Gerth & Glagolevskij (2000) and applied to real stellar objects by Gerth et al. (1998, 1999, 2000), Gerth & Glagolevskij (2001) and Glagolevskij, Gerth (2000, 2001, 2002).

The *magnetic charges* are treated analogously to the *electric charges*. This is advantageous for the computation of the spherical field of a point-like source by the standard algorithm of the differential operator *gradient*.

The idea of construction of the stellar magnetic field out of its generating sources was adopted by Khalack et al. (2001, 2002, 2003), who came to the same conclusions as we had done before.

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2 Modeling by the MCD-method

The modeling method of the "*Magnetic Charges*" gives a description of the magnetic vector field on the star's surface outside and inside the star. So it would be possible to trace the lines of force back from the surface somewhat into the interior, keeping in mind, however, the limitations of the model in the vicinity of the sources. Sources outside the star, like orbiting magnetic bodies of a binary system, can affect the atmosphere of the main star by their magnetic field.

The MCD-method relates to single field sources with "magnetic charges", which are surrounded by spherical magnetic fields. Such sources might be distributed anywhere in space. A spherical body, like a star, shows to the observer its magneto-informative surface, which is penetrated by the lines of force. Since single *magnetic charges* do not exist in reality, we take them as *virtual sources* — by analogy to the virtual light source in optics, from which the light seems to diverge (after a mirror or a lense). A couple of two oppositely charged sources, however, physically form a *magnetic dipole*. The fields of numerous dipoles superpose linearly on irregularly arranged *quadrupoles* and *multipoles* rendering all possible field structures. Such multipoles are arrangements of field sources and should not be mistaken as a derivation of the *Legendre* coefficients in a series of spherical harmonic functions.

The vectorial field of a point-like source is calculated for any point in the surrounding space and, therefore, for any geometrical body like the sphere of a star, — in contrast to the calculation of the field by means of spherical harmonics (see Oetken 1977, Bagnulo et al. 2000, 2001a,b), — which relates only to the surface of a sphere. This means that the calculation of a decentered dipole or a dipole located outside the star cannot be performed by the method of spherical harmonics.

A special standard algorithm calculates the field strength as the gradient of the potential of the charged source — making dispensable the use of *Legendre's* spherical functions for MCD-modeling of stellar magnetic fields.

3 The observational results used for modeling

The phase relations of the effective (mean longitudinal) magnetic field strength B_e and the surface (mean field modulus) magnetic field strength B_s taken by some authors differ heavily. This is shown in the paper of Leone & Catanzaro (2001), where different phase curves are drawn together. The curves show systematic deviations up to 1 kG, although the intrinsic accuracy of the measurements by any of the authors is essentially higher. Thus, one has to be cautious to compile simply the data. We relate the modeling only to homogeneous data sets.

From all available observational data we chose the phase (P) relations $B_s(P)$ from Wolff & Wolff (1970) and Mathys et al. (1997) (Fig. 1b, 2a). The first data are taken from old photographic observations by the lines of Cr, Ti, and Fe. The second data were obtained by the line of Fe II (6149.2 Å) with a CCD-camera. Further we employ the phase curves $B_e(P)$ with magnetic field measurements of the hydrogen line H_β by Borra & Landstreet (1980) (Fig. 1a, 2b). A third data set was taken by Wade et al. (2000) using Fe-lines, which will be analyzed separately and compared with the other results (Fig. 2c).

All observational phase curves are arranged using the ephemeris after Kurtz (1989):

$$\mathrm{JD}_{magnetic\ max} = 2434204.70 + 18.4868\ \mathrm{E}.$$

Relating to the representation of the observational data in Fig. 1 and Fig. 2, let us consider some properties of the phase relations that have an effect on the model of the magnetic field in the case of βCrB. The small amplitude of $B_s(P)$ leads to a small angle of declination i. On the other hand, the angle i has an effect on the amplitude of the calculated relation $B_e(P)$. The inclination angle $i = 13°$ found by us corresponds best of all to both phase relations. The small magnitude of $B_e < 1$ kG together with the large quantity $B_s \approx 5.5$ kG argues for the assumption that the magnetic poles are located at the limb of the visible disk, e.g., close to the equatorial plane. The star is visible almost from its pole.

3.1 The dipole model

In order to construct a magnetic dipole, the position of the two "magnetic charges" of opposite polarity inside the star is defined by a procedure of successive approximations.

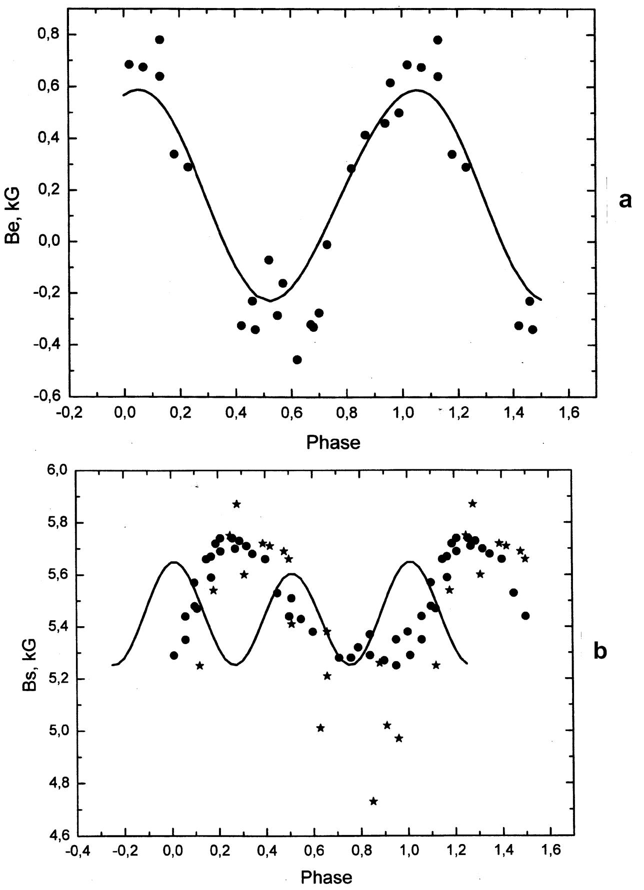

Figure 1: *Phase relations for βCrB, applying the dipole model (solid lines — calculated phase curves): a) effective magnetic field B_e (Borra & Landstreet 1980); b) averaged magnetic field on the surface B_s, asterisks — data from Wolff & Wolff (1970), dots — data from Mathys (1997).*

At first, we select arbitrarily a magnetic moment $M = Qr$ (Q — magnetic charge, r — its distance from the center of the star as fraction of radius), the longitude λ, the latitude δ, and the inclination i to the line of sight. For the centered dipole model with positive (+) and negative (–) charges at equal distances of the sources from the center yields:

$r_+ = r_-$; $\lambda_+ - \lambda_- = 180^o$; $\delta_+ = -\,\delta_-$.

The fitting of the observed to the calculated phase relations $B_e(P)$ and $B_s(P)$ is carried out by the least-squares optimization method. It comes out that the position of the positive charge in βCrB is $\lambda = 0°$, $\delta = +l°$, and that of the negative charge is $\lambda = 180°$, $\delta = -1°$. The maximum field strength at the poles is calculated for the dipole model to be $B_p = 9.4$ kG. The phase curves derived from the model are shown in Fig. 1 by solid lines. In principle, the dipole model requires two maxima of B_s, which should pass through the central meridian of the positive and negative magnetic poles. But we see that the maximum and minimum of the relation $B_s(P)$ appear at the moment when $B_e \approx 0$, instead of the moment when B_e has its maximum. This consideration and the test calculation show that the structure of the magnetic field of βCrB cannot be dipolar one — as it was already asserted by Bagnulo (2000a,b), who demanded addition of a non-linear quadrupole to represent the surface magnetic field of βCrB.

The mode1ing of some other stars, for example, HD 126515 (Glagolevskij & Gerth 2000), has proven that such a situation is possible in the case of arrangement of two decentered dipoles. But for βCrB a much more complex situation turns out.

The model of a decentered dipole results in an amplitude of B_s larger than the observed one. By the method of iterative approximation we found that both observed curves can be explained assuming the presence of two oppositely directed dipoles, whose magnetic charges are located at a distance of 0.3 stellar radius from the

Table 1:

No	q	r	λ	δ
1	-1	0.3	351	-6.5
2	$+1$	0.3	12	0
3	-1	0.3	176	5.3
4	$+1$	0.3	196	0

center of the star (see Fig. 3). We call such a combination of two arbitrarily arranged dipoles an *irregular quadrupole*.

3.2 The quadrupole model

The disagreement between the computed and the observed phase curves underlying the dipole model shows that βCrB has a magnetic field of complicated structure.

The model, comprising the B_s and B_e data from Fig. 1a and Fig. 1b, is calculated using the following parameters:

All charges have the same relative value $q = 1$, which is fitted to the observed magnetic field strength on the surface. The magnitude of the magnetic field equals 14.5 kG on the magnetic poles. The distance of the charges from the center is $r = 0.3$ the stellar radius. We see in Fig. 3 that both dipoles are arranged more or less in the equatorial plane symmetrically about the center of the star. The inclination angle has been determined by variation of parameters and iterative approximation as $i = 13°$.

For comparison we add in Fig. 2 more recent observational data from Wade et al. (2000), which are presented in Fig. 2c. The fitting of the curve could be achieved by a minor variation of the parameter δ ($\delta_1 = -6.6$, $\delta_3 = 5.0$), the effect of which proves to be very sensitive.

The distribution of the magnetic field strength over the surface is represented in Fig. 4. The double dipole model describes well the common configuration of the magnetic field, but in detail there may be some differences. So we see in Fig. 2b a small peak in the negative extremum of the observed phase curve $B_e(P)$, which is smoothed in the calculated curve. In the paper of Wade et al. (2000) a relation $B_e(P)$ without such a peak is given, obviously because of the more accurate measurements carried out by new observation and reduction techniques (Fig. 2c). The smoothed phase curves in Fig. 2b and Fig. 2c deviate a little by the amplitude, the reason for which is not known yet (maybe an instrumental effect). In Fig. 2a, representing the function $B_s(P)$, the observations show a broad minimum, which is not contained in the calculated relation. However, the measurements of Wolff & Wolff (1970), marked in Fig. 2a by asterisks, contradict this assertion. Those differences have to be attributed to the uncertainties of the different sets of measurements.

Some significant differences of the curves drawn after the model and the ones obtained by observation can be explained by two reasons:
1) the relation $B_s(P)$ has been derived from the metallic lines, which are distributed inhomogeneously;
2) the magnetic field possesses a more detailed fine structure than the one we have obtained.

The first reason seems to be more probable. The phase curve $B_s(P)$ in Fig. 2a has a maximum at phase 0.25 with $B_e \approx 0$. Such a situation may occur only in the case, where the region between the oppositely arranged poles passes through the meridian and the magnetic charges are located under a small angle λ of longitude.

The configuration found for βCrB consists obviously of two oppositely directed dipoles. Hereby, the axes of the dipoles are not strictly parallel. The large distance between the dipoles of 0.6 the stellar radius confirms the idea of existence of two individual dipoles, which combine to a quadrupole. Assuming other distances of the dipoles from the center, the fit between the calculated and the observed phase relations becomes worse.

The phase relation could still be influenced by other phenomena. Thus, in the papers of Wolff (1978) and Romanyuk (1986) some characteristics of the star βCrB have already been mentioned, in particular, the phase shift of the magnetic curves derived from the lines before and after the Balmer jump.

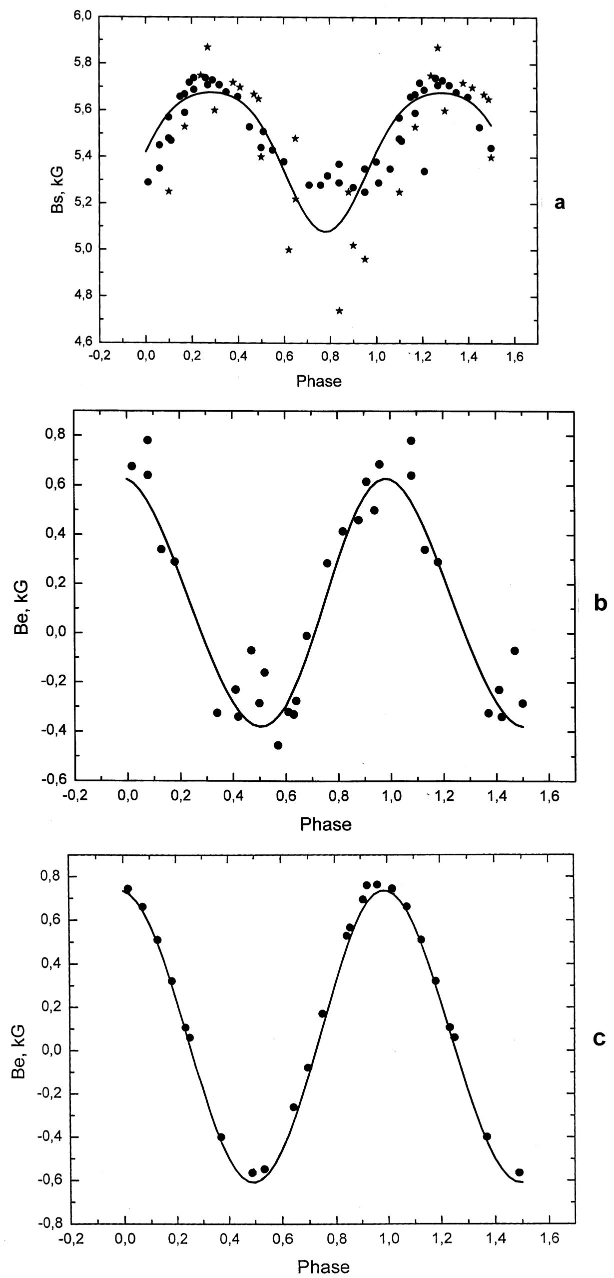

Figure 2: *Phase relations for βCrB, using the quadrupole model (dots — observations, solid lines — calculated phase curves): a) averaged magnetic field of βCrB on the surface B_s, asterisks — data from Wolff & Wolff (1970), dots — data from Mathys et al. (1997); b) effective magnetic field B_e (Borra & Landstreet, 1980); c) effective magnetic field B_e (Wade et al. 2000).*

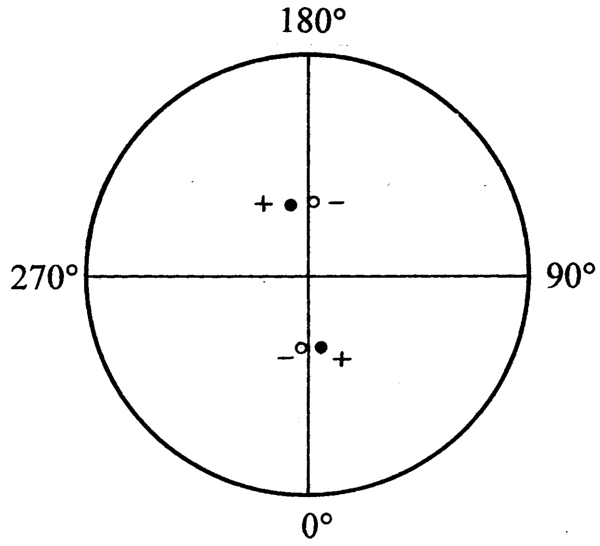

Figure 3: *Scheme of the distribution of the magnetic dipoles inside the star (view from the pole).*

4 About the gradient of the magnetic field

Our model does not predict a significant gradient of the field as a function of depth. If we assume a thickness of the atmosphere of 10^3 km, then the maximal difference of the surface magnetic field at the extreme levels amounts to 25 G. This has been calculated with used program, inserting the radii on the surface and the radius 1000 km higher.

The change of the field structure with depth, starting from the surface, goes at first rather slowly, as we can see in Fig. 5. This result does not contradict a suggestion given by Wolff (1978), who assumes a significant gradient of the magnetic field with depth. The slow course changes strikingly at 0.3 the stellar radius, where the sources are located at points of singularity. In the infinitesimal vicinity of the singularities the field strength rises to infinity. Closer to the center the field strength changes the polarity, but has a similar structure on an inner sphere at 0.1 R.

This is correct for real point-like sources, as in case of electrical charges. However, the analogy between electrical and magnetic charges has its limitations. The closed magnetic lines of force have no singularity at all. Therefore, the inner structure of the magnetic field is not simply founded in a few magnetic sources. Nevertheless, it demonstrates the continuity of the magnetic field transition through every shell of the sphere embedded in the surrounding space.

The *magnetic charges* as *virtual sources* render a powerful heuristic approach for the calculation of the magnetic field structure, which gives an advantageous base for programming and fits the better to the reality the more the distance from the singularities increases.

5 Conclusion

We draw the conclusion that our model — as well as stated in the paper of Bagnulo et al. (2000) — yields a small angle of the rotation axis to the line of sight ($\approx$13°) and a large angle ($\approx$90°) to the plane spanned by the dipoles. With the construction of an irregular quadrupole model the contradiction of the calculated B_e- and B_s-curves could be overcome.

At the present time, the hypothesis of relic origin of stellar magnetism in CP stars is commonly supported. However, in this case the structure of a field similar to that of βCrB is hardly possible. Such a structure might easier be explained by the hypothesis of a magnetic dynamo. If we locate this star on evolutionary tracks, then we can see that its way passes the early stages of the evolution through regions occupied by TTau stars; that means, the star could have passed in its early stages a phase of convective instability, when

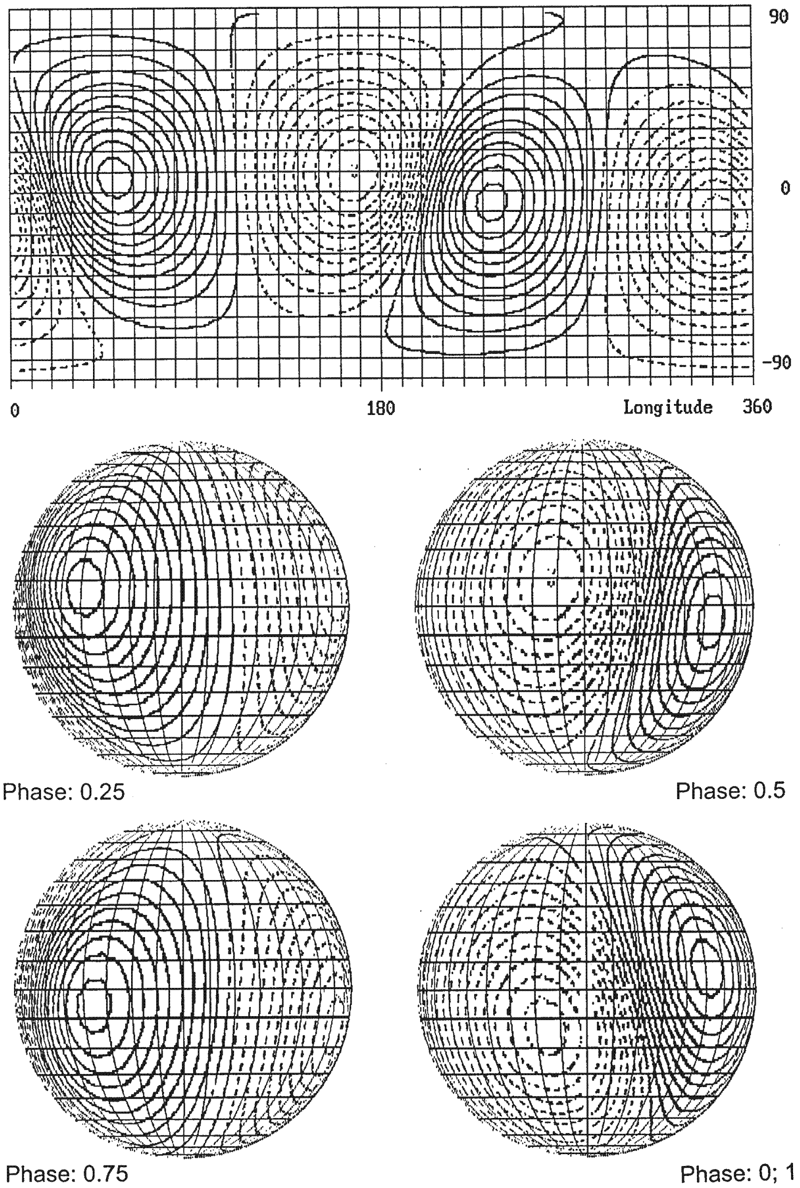

Figure 4: *Distribution of the magnetic field strength over the surface represented by iso-magnetic lines of the radial magnetic field strength. Division into 20 equal steps between the positive and the negative poles with the polar strength of* $B_p = 14.5$ *kG. Top: Pseudo-Mercator map with four magnetic extrema of alternative polarity. Bottom: Globes for four rotation phases, solid lines — positive regions; dashed lines — negative regions.*

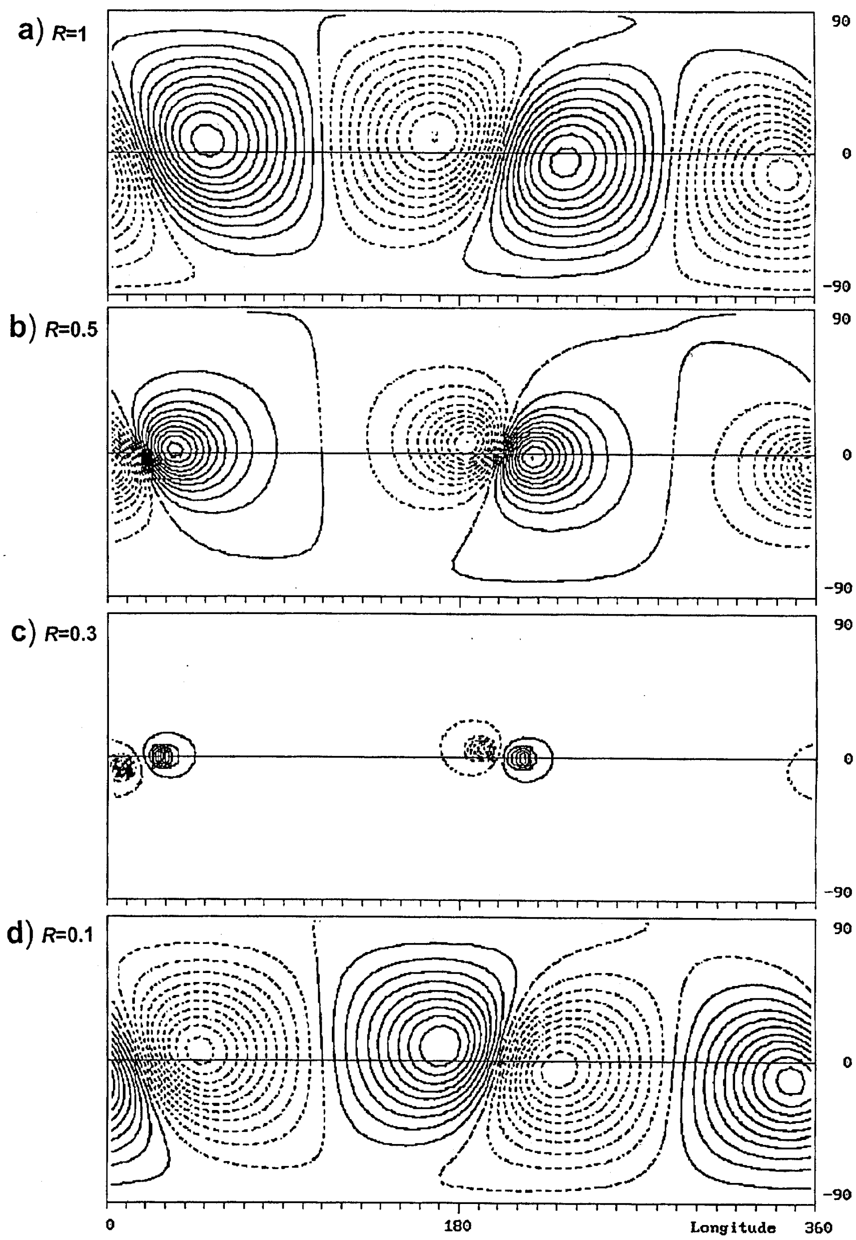

Figure 5: *Distribution of the field strength by 4 point-like sources inside the star at different levels (radius) under the surface. The iso-magnetic lines characterize the map on each radial shell between the (internally computed) maximum and minimum of the field strength. Difference divided in 20 steps, solid lines — positive regions, dashed lines — negative regions (computer program written by E. Gerth).*

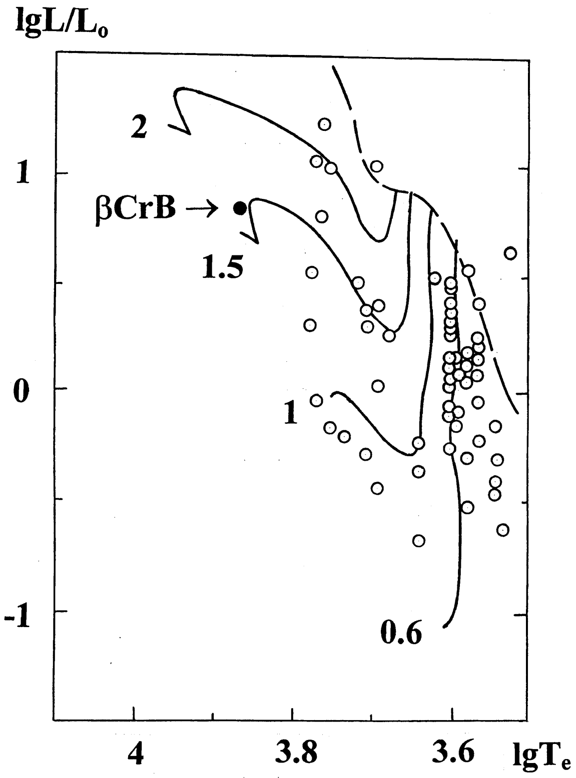

Figure 6: *Location of young stars of type TTau (white circles) and of βCrB (black circle) in the Hertzsprung-Russell diagram.*

a magnetic field could have been generated. In this case we do not observe in βCrB the original relic field, but a secondary one, generated by the effect of a stellar dynamo. In Fig. 6 the Hertzsprung-Russell diagram, taken from the paper of Palla & Stahler (1994), is demonstrated, in which the evolutionary tracks of early stars, TTau stars, and that of βCrB are drawn. From this diagram we can see too that, obviously, all CP stars with masses $M < 2M_{\odot}$ ($T_e \approx 9000$ K), i.e. SrCrEu type stars, went in the past through the convective phase.

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