Magnetic stars, 2004, 166-174

Phase curves and line profiles of stellar magnetic fields related to the Stokes parameters I, Q, U, V

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Abstract. The phase curves of the integral magnetic field strength and the phase-dependent line profiles in the four polarization modes are calculated and graphically demonstrated using the method of modeling magnetic field structures on the surface of a star by the *Magnetic Charge Distribution* (MCD). Phase curves and line profiles are analytically represented by convolution integrals, which allow the linear superposition of elementary relations derived from single magnetic sources as the generating constituents of any complex magnetic field. Usually, the influence of the geometrical conditions of vision on the line profile, basically shaped by radiative transfer through the stellar atmosphere, has been neglected. The magnetic field distribution over the surface strongly affects the shape of the line profiles. It leads to asymmetric profiles with high uncertainty in practical measurement of magnetic fields by Zeeman displacement of spectral lines.

Key words: stars: magnetic fields – methods: numerical – line: profiles

1 Introduction

Modeling of the magnetic field structure of stars for fitting the derived magnitudes of the field strength to the observation requires a physically founded theory for the construction of the field by the generating magnitudes. Relating to the potential theory, the magnetic field — like every vector field — is generated by its sources and vortices, which combine superposing the fields linearly. The modeling method of the Magnetic Charge Distribution (MCD) after Gerth et al. (1997, 2000) allows the calculation of the fields of point-like sources with virtual magnetic charges. The combination of two oppositely charged sources is a magnetic dipole with a magnetic moment, which is a real generating magnitude and can be taken as the elementary brick for construction of any complex magnetic field. The observation of the integral magnetic field from the star's surface is given as phase curves and line profiles of the polarized light according to the four Stokes parameters I, Q, U, V. With the help of the model, typical phase curves and line profiles can be constructed, compared with the observed ones, and fitted to them by variation of parameters.

The theory for the algorithms of the computer program is described by Gerth et al. (1997) and Gerth, Glagolevskij (2001). The program allows the calculation and the graphical representation of maps, globes and line profiles in connection with the phase relation. The line profiles with the center of gravity are shown on the screen, moving in the course of the star's rotation.

2 Observation of the integral radiation

On the stellar surface, the field is distributed by the two coordinates: φ — longitude and δ — latitude, arranged as a matrix in a Mercator map. The distribution function $B(\delta, \varphi)$ of the map can be taken from observation or from calculation of star models. Bagnulo et al. (2001) calculate the surface distribution of the magnetic field by means of spherical harmonics. We relate to a model, which is based on virtual magnetic sources, which combine to real magnetic aggregates.

The spatial field distribution of a *virtual magnetic monopole* is the elementary field, for the calculation of which a standard algorithm is developed (Gerth and Glagolevskij 2001). This is the heuristic reason for the use of virtual sources.

The field components of a decentered magnetic monopole with r as the distance of the point-like source to the center and the magnetic charge Q in the constant C = – $Q/4\pi$ are:

$$\boldsymbol{B}_r = (C/r^3)[\cos\delta(\cos\varphi + \sin\varphi) + \sin\delta], \tag{1}$$

$$\boldsymbol{B}_\varphi = (aC/r^3)\cos\delta(\cos\varphi - \sin\varphi), \tag{2}$$

$$\boldsymbol{B}_\delta = (aC/r^3)[\cos\delta - \sin\delta(\sin\varphi + \cos\varphi)]. \tag{3}$$

The calculation of the magnetic field strength renders such a triple of values to every point of the surrounding space. The visibility of such a point on the surface depends on a lot of conditions bound to geometry, phase and physics of the star. The globe of the star is seen by the observer under different aspects, caused by its rotation and the inclination i to the rotational axis. Besides of this, the visible disk is “vignetted” by the limb darkening according to the empirical formula with ε denoting the angle from the center of the disk

$$k = 1 - 0.4\cos\varepsilon. \tag{4}$$

2.1 The phase relation

For the visibility of the star by the observer, we define a window function $w(i, \varepsilon, \delta, \varphi)$, containing the inclination i, the projection of each surface element to the line of sight, and the limb darkening with its angular distance ε from the center of the visible disk, which averages and normalizes the vector $\boldsymbol{B}(r, \delta, \varphi)$ with the orthogonal components B_r, B_f, B_d of the surface magnetic field:

$$\boldsymbol{B}_{\text{int}}(t) = \frac{\int\limits_{\delta=-\pi/2}^{\pi/2}\int\limits_{\varphi=0}^{2\pi} \boldsymbol{B}(r, \delta, \varphi) w(i, \varepsilon, \delta, \varphi - t)\mathrm{d}\varphi\mathrm{d}\delta}{\int\limits_{\delta=-\pi/2}^{\pi/2}\int\limits_{\varphi=0}^{2\pi} w(i, \varepsilon, \delta, \varphi - t)\mathrm{d}\varphi\mathrm{d}\delta}. \tag{5}$$

Equation (5) is the analytical representation of the phase curve for three orthogonal vectors in Cartesian space.

This integral formula gives the integral mean of the disk seen by the observer and comprises the *convolution integral*, which represents the rotation of the star with its map $B(\varphi, \delta)$ behind the window $w(i, \varepsilon, \delta, \varphi)$. The magnitude t is set equivalent to the longitude φ and characterizes the rotation at the time of the momentary orientation angle at the longitude φ as a function of time. The denominator makes the normalization.

2.2 The line profile

Usually we measure the (integral) magnetic field from the *Zeeman displacement* of the line profiles of oppositely circularly polarized light. What we call the “*effective magnetic field*” B_{eff} is not a mean value but already the result of weighting and convolution of the radiation flux containing the magnetic field information about the form and the position of the profiles of all surface elements, in which the line forming process takes place.

The observed line profile is formed by different physical processes:
1) radiative transfer with atomic interaction in the stellar atmosphere;
2) microturbulence (thermic motion);
3) macroturbulence (convection);
4) flow movement (circular, meridional, and equatorial currents);
5) observational and instrumental conditions (seeing, optics of telescope and spectrograph, resolution of storage, reduction etc.);
6) rotation (including inclined and differential rotation);
7) geometry of the radiating parts of the star’s surface seen as a disk by the observer in integral light.

All these processes lead to a vast mixing of influences. As far as convolution is involved, we can analyze single broadening profiles, like the so-called *instrumental profile*. We put all profiles, which broaden the *radiative transfer profile,* together with the profile function ω, which comprises more or less the influences 1–5. The point 6 is connected with the radial velocity, which gives a shift of the line profile in the spectrum and has to be treated equally to Zeeman displacement due to the magnetic field.

In this paper emphasis is laid on the iinfluence of the geometry according to point 7. Hitherto, the profile formation has been investigated mainly at plane atmosphere layers, not accounting for the spherical star body. In order to get clear relations, we restrict the line profile formation to the geometrical conditions, which we will analyze separately from all other influences. The basic profile, of course, is formed by the transfer process of radiation through the stellar atmosphere, to which the other influences are subordinated.

The distribution of the polarized radiation over a region b around $\boldsymbol{B}$ is defined by $\omega(b)$ and convoluted to phase integral equation (5):

$$\boldsymbol{B}_{\text{int}}(t,b) = \frac{\int\limits_{\delta=-\pi/2}^{\pi/2} \int\limits_{\varphi=0}^{2\pi} \int\limits_{-\infty}^{+\infty} \boldsymbol{B}(r,\delta,\varphi,\beta) w(i,\varepsilon,\delta,\varphi-t)\omega(\beta-b)\mathrm{d}\beta\mathrm{d}\varphi\mathrm{d}\delta}{\int\limits_{\delta=-\pi/2}^{\pi/2} \int\limits_{\varphi=0}^{2\pi} \int\limits_{\infty}^{+\infty} w(i,\varepsilon,\delta,\varphi-t)\omega(\beta-b)\mathrm{d}\beta\mathrm{d}\varphi\mathrm{d}\delta}. \tag{6}$$

Equation (6) is the analytical representation of the geometric line profile in the course of phase, containing the convolutions due to rotation and the frequency distribution of the field strength.

In our computing program we relate to the fact that the gravity center of two profiles of different height and position is given by the mean of the centers weighted by the profile integrals. Thus, we weight the magnetic field vector, projected onto the line of sight, of all surface elements with their spherical projection and limb darkening and integrate them over the visible hemisphere. If from the visible disk the majority of field vectors is directed to the observer, then there would occur a high maximum with a narrow line width.

For the investigation of the *geometrical profile* for its own — without the other line forming influences — we set the distribution function as a δ-function

$$\omega(b) = \delta(\beta - b). \tag{7}$$

3 The Stokes parameters I, Q, U, V

The four polarization modes of light radiation of a star viewed under the inclination angle i to the rotation axis are described after Stokes by the parameters:
1) I — intensity of the entire radiation flux;
2) Q — linear polarization in the plane to the rotation axis;
3) U — linear polarization perpendicular to the rotation axis;
4) V — circular polarization to the line of sight.

The four Stokes parameters constitute a linear set of quantities, which describe the polarization conditions of a radiation beam completely. If we take the four parameters as a vector, then changing conditions of spatial arrangement and absorption can be easily accounted for by linear transformation by a quadratic matrix of rank 4. By this way, the polarization modes were calculated in the papers of Piskunov and Kochukhov (2002). We use here a more direct way of calculation on the basis of vector algebra.

The polarization modes Q, U, and V of the magnetic field follow from the projection of magnetic field vector on the line of sight in 3 orthogonal directions. As outlined by Gerth and Glagolevskij (2001), the radial direction of the field vector in every element on the surface is given in Cartesian coordinates with the unity vectors **i**, **j**, **k** and the geographical coordinates of the longitude φ and the latitude δ to

$$\mathbf{a}_r = \cos\delta\cos\varphi\mathbf{i} + \cos\delta\sin\varphi\mathbf{j} + \sin\delta\mathbf{k}, \tag{8}$$

$$\mathbf{a}_\varphi = -\cos\delta\sin\varphi\mathbf{i} + \cos\delta\cos\varphi\mathbf{j}, \tag{9}$$

$$\mathbf{a}_\delta = -\sin\delta\cos\varphi\mathbf{i} - \sin\delta\sin\varphi\mathbf{j} + \cos\delta\mathbf{k}. \tag{10}$$

With the three spherical components B_r, B_φ, and B_δ of the magnetic field vector at the surface of the star is given in Cartesian coordinates:

$$\boldsymbol{B} = B_r\mathbf{a}_r + B_\varphi\mathbf{a}_\varphi + B_\delta\mathbf{a}_\delta. \tag{11}$$

The projection of the magnetic field vector related to each point of the surface is carried out by a scalar multiplication of the magnetic field vector $\boldsymbol{B}$ with its components B_a, B_φ, and B_δ adjusted to the vector of the line of sight,

$$\mathbf{q} = \sin i \sin t\mathbf{i} - \cos i\mathbf{j} + \sin i \cos t\mathbf{k}, \tag{12}$$

$$\mathbf{u} = -\cos i\mathbf{i} + \sin i \sin t\mathbf{j} + \sin i \cos t\mathbf{k}, \tag{13}$$

$$\mathbf{v} = \sin i \cos t\mathbf{i} + \sin t \sin t\mathbf{j} - cosi\mathbf{k}. \tag{14}$$

Carrying out the scalar multiplications, the polarization modes Q, U, V are:

$$\begin{aligned} B_\mathrm{Q} = \boldsymbol{B}\ \mathbf{q} = {} & B_r(\cos\delta\cos\varphi\sin i\sin t - \cos\delta\sin\varphi\cos i + \sin\delta\sin i\cos t) + \\ & + B_\varphi(-\cos\delta\sin\varphi\sin i\sin t - \cos\delta\cos\varphi\cos i) + \\ & + B_\delta(-\sin\delta\cos\varphi\sin i\sin t + \sin\delta\sin\varphi\cos i + \cos\delta\sin i\cos t). \end{aligned} \tag{15}$$

$$\begin{aligned} B_\mathrm{U} = \boldsymbol{B}\ \mathbf{u} = {} & B_r(-\cos\delta\cos\varphi\cos i + cos\delta\sin\varphi\sin i\sin t + \sin\delta\sin i\cos t) + \\ & + B_\varphi(\cos\delta\sin\varphi\cos i + cos\delta\cos\varphi\sin i\sin t) + \\ & + B_\delta(\sin\delta\cos\varphi\cos i - sin\delta\sin\varphi\sin i\sin t + \cos\delta\sin i\cos t). \end{aligned} \tag{16}$$

$$\begin{aligned} B_\mathrm{V} = \boldsymbol{B}\ \mathbf{v} = {} & B_r[\cos\delta\sin i(\cos\varphi\cos t + \sin\varphi\sin t) - \sin\delta\cos i] + \\ & + B_\varphi[\cos\delta\sin i(\cos\varphi\sin t - \sin\varphi\cos t)] + \\ & + B_\delta[-\sin\delta\sin i(\cos\varphi\cos t + \sin\varphi\sin t) - \cos\delta\cos i]. \end{aligned} \tag{17}$$

From the modes Q, U, V we derive the Stokes parameter I as the quadratic sum

$$B_\mathrm{I} = \sqrt{B_\mathrm{Q}^2 + B_\mathrm{U}^2 + B_\mathrm{V}^2}. \tag{18}$$

For the numerical calculation we replace the integral transformations by matrix multiplication. The map of the magnetic surface field is discretised into surface areas as matrix elements, each element representing the integral mean value of this area. The profile is compiled in a frequency distribution as a grid of classes, in which the values of the magnetic field strength of every surface element is sorted in. The computation is performed for each vector component separately.

4 Phase curves and line profiles

We demonstrate the computation of phase curves and line profiles by graphics. Here are raised only two typical cases:

1) magnetic monopole in the equatorial plane,

2) magnetic dipole, both seen equator-on in the equatorial plane.

The magnetic dipole in the plane of the equator is realized approximately in the CP star 53 Cam (Bagnulo et al. 2000, Gerth et al. 2000). The deviation of the phase curve from the sinusoidal form is caused by the arrangements of rings with accretion or depletion of chemical elements around the poles.
We demonstrate here only the very simple cases. The algorithm for the calculation of phase curves and line profiles from the magnetic field distribution on the star's surface is, of course, much more general and allows arbitrary angles of sight and the overlay of an opacity layer on the surface.

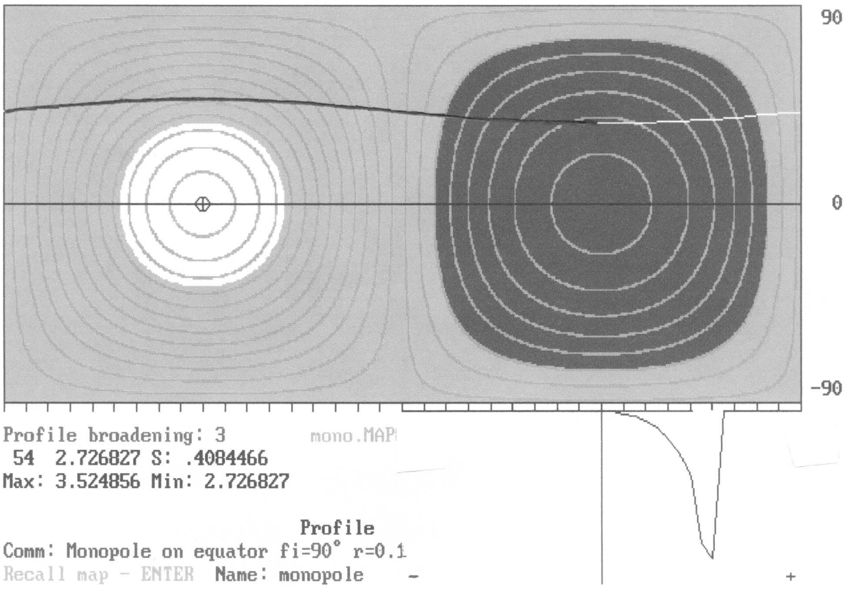

Fig. 1.*Magnetic monopole with coordinates φ = 90°, δ = 0°, r = 0.5. The field is always positive with a minimum at φ = 270°, δ = 0°. The line profile is asymmetric and has its deepest point at the field minimum by phase 0.75.*
Computer screen demonstration: Profile moving in phase of rotation.

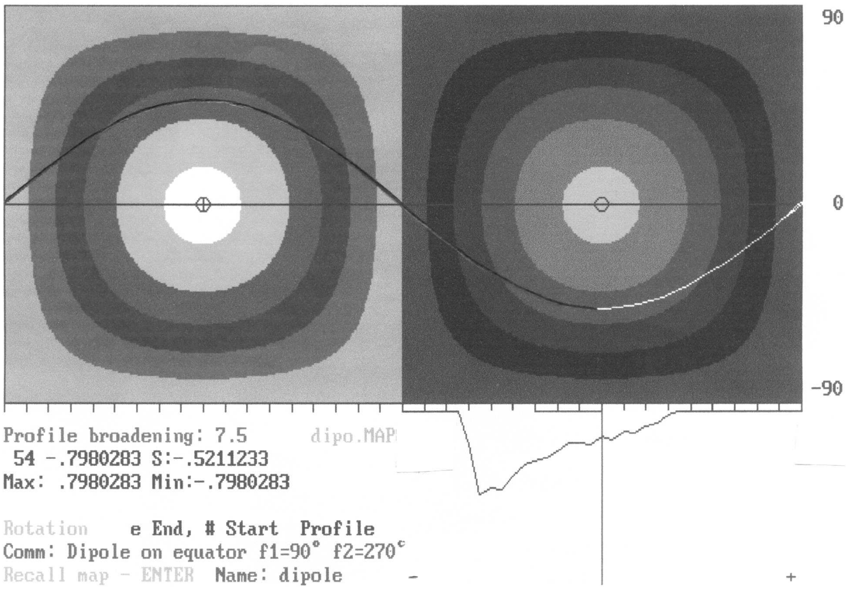

Fig. 2. *Central magnetic dipole with separated magnetic charges:*

Radius-fraction	Longitude	Latitude	Charge
$r_1 = 0.5$	$\varphi_1 = 90^o$	$\delta_1 = +45^o$	$Q_1 = +1$
$r_2 = 0.5$	$\varphi_2 = 270^o$	$\delta_2 = -45^o$	$Q_2 = -1$

The line profile changes the polarity at phases 0 and 0.5 with a nearly rectangular form and shows at the poles an extreme asymmetry at phase 0.75 with a steep edge at the side turned away from the middle line. (Screen demonstration).

Fig. 1 and Fig. 2 are images taken from the monitor screen, comprising the magnetic field map with the phase curve of the integral magnetic field (Stokes V). The profile is shown at phase 0.75, marked by the change of the phase curve from black to white.

The phase curves of the monopole and the dipole fields as background structure are put together for all modes I, Q, U, V in Figs. 3 and 4.

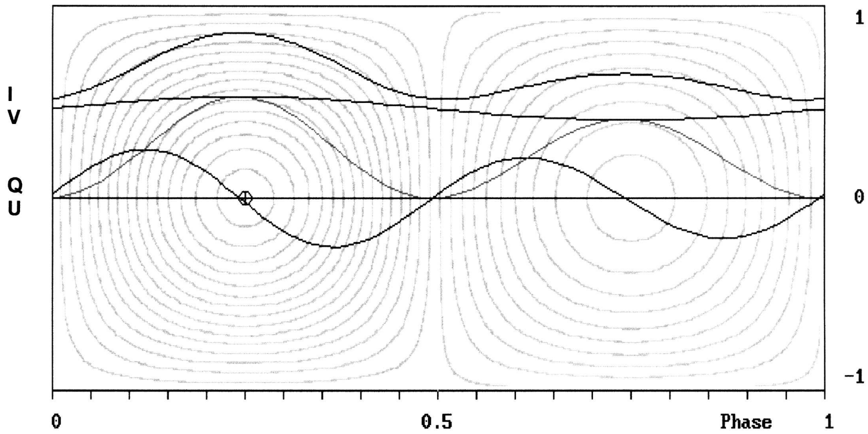

Fig. 3. *Phase curves of Stokes I, Q, U, I for a magnetic monopole.*

The curves of the parameters I and V show a similar behavior for the overall positive field strength. The Q-curve has positive and negative parts because of the gradient of the field in direction of the longitude with maxima and minima and zero at the pole and counter pole. The U-curve is always positive.

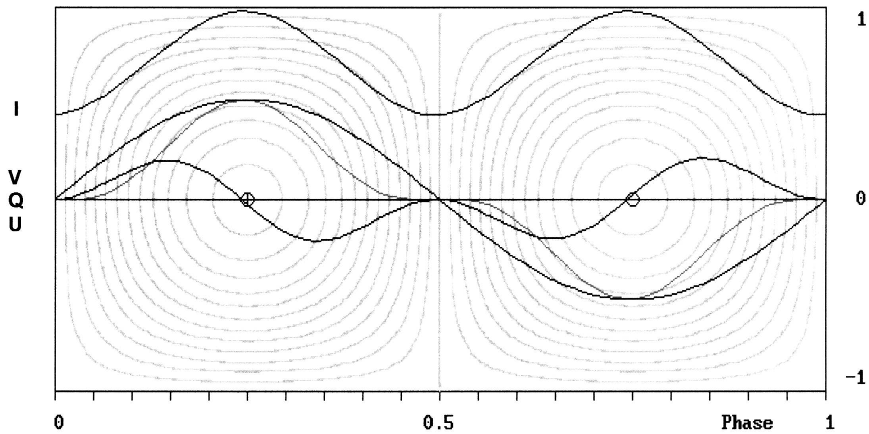

Fig. 4. *Phase curves of Stokes I, Q, U, I for a magnetic dipole.*

The first part of the dipole phase curves up to phase 0.5 shows principle agreement with the curves of the magnetic monopole. In the second part the polarity is changing. Extrema and zero points are similarly arranged. The U-curve has its extrema at the poles, which coincide there with the V-curve. The I- and Q-curves are mirror-image-like symmetrical to phase 0.5, whereas the Q- and V-curves show rotational symmetry with turning point at zero.

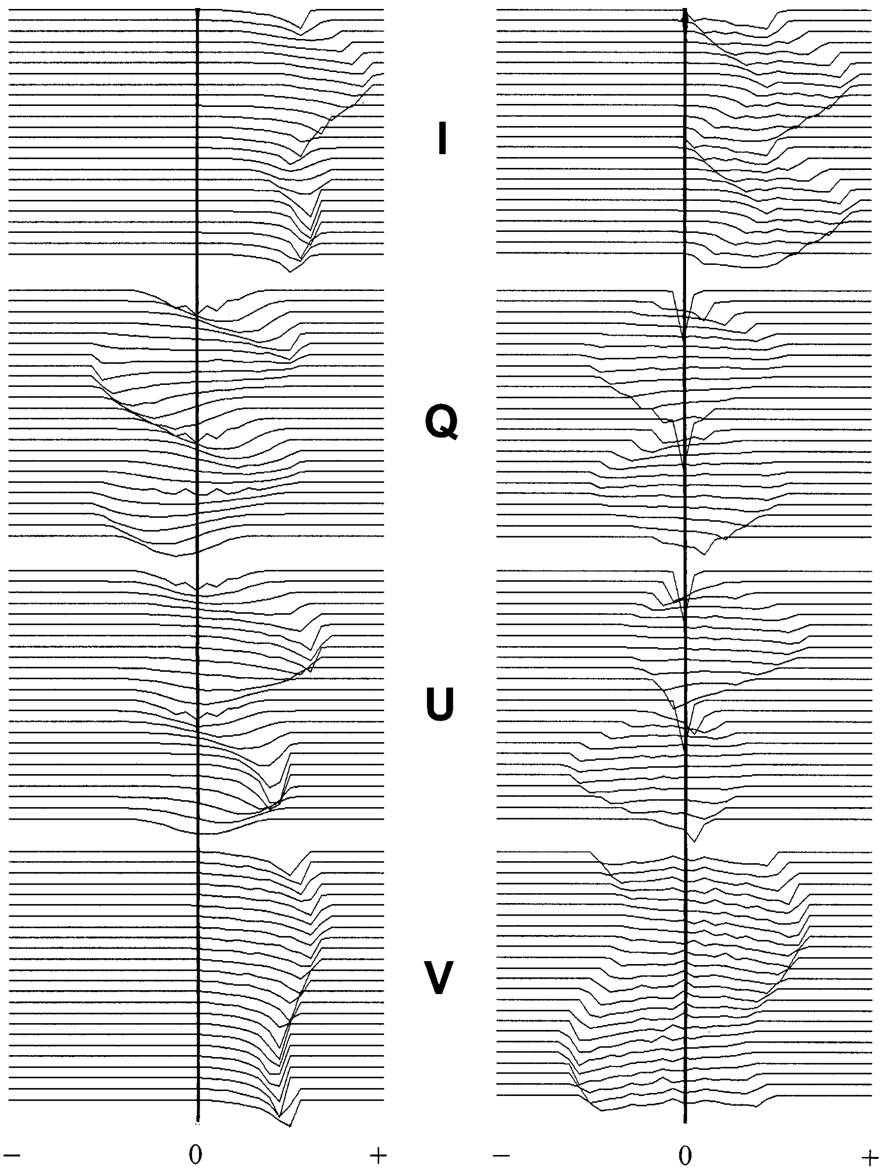

Fig. 5.*Series of geometrical line profiles for Stokes I, Q, U, V.*
Abscissa: Definition range of frequency distribution.
Ordinate: phase 0 · · · 1, phase step 0.05.
Left panel: A virtual magnetic monopole is investigated for its Stokes profiles. The parameters I and V reflect best the expected course of the field; Q shows a double wave with change of polarity; U is only positive; I, U an V have the steep edge at the positive side. The line profiles are asymmetric and vary heavily with the phase.
Right panel: Stokes profiles of a magnetic dipole consisting of two magnetic charges in the equatorial plane. The parameters I and V reflect also for a dipole best the expected course of the field. Besides the bumping-like double wave, at Q and U the variability of the wave length with a change of polarity and asymmetry is conspicuous.

5 Superposition of elementary phase curves and line profiles

The theoretical essence of the modeling method of magnetic fields by sources and vortices is the reduction to elementary units and the composition to big complexes by linear superposition. This holds for the magnetic field itself, as outlined by Gerth et al. (2000). However, also the magnitudes and relations derived from the magnetic field have the same property of superposability — as far as the integrals in equations (5) and (6) are distributive like the summands of a sum, which is fulfilled for the integration of the radiation in view by

the observer over the surface elements.
The convolution in these equations (5) and (6) makes the form of the phase curve with the line profile by the distribution function ω. Both, phase curve and line profile, are closely connected: the phase curve is the periodic variation of the center of gravity of the line profile in the course of phase.

We see in Figs. 3 and 4 that the left part of the four Stokes phase curves look very similar. If we put such curves of a positive monopole at $\varphi = 90°$ as a negative monopole on $\varphi = 270°$, then we get the dipole phase curves in Fig. 4 by addition. The monopole phase curve can be taken as the elementary one. But also Fig. 5 shows that the geometrical profiles of the four Stokes parameters are additive in the same manner, if we use elementary monopole profiles.

The addition of elementary phase curves and profiles can comprise larger combinations, such as a dipole with a positive and a negative magnetic charge. A precondition of the superposition is that all elementary units belong to the same inclination angle i — being obvious for a definite star.

Last but not least, there is still another important superposition of elementary units: the composition of effective line profiles out of the constituents of a Zeeman pattern. There are two possibilties:
1. Composition of the distribution function $\omega(b)$ coordinated to the δ-functions with the corresponding Landé-factor and convolution with the profile function physically originated in the stellar atmosphere.
2. Total calculation of the field structure with phase curves and geometric profiles out of the sources and summation with a shift in the spectrum coordinated to the Landé-factors, respectively, the effective z-values.

The superposition can be extended to a spectral region with different lines, enabling the calculation of synthetic spectra.

6 The importance of the geometrically caused asymmetry of the line profiles for the magnetic field measurement

The geometrical line profiles are usually asymmetric and deviate heavily from the normal (Gauss)distribution. The maximum and the center of gravity do not coincide. In contrary to plausible imagination, a maximum of the integral magnetic field strength is not given by the sight pole-on, but by the highest quantity of the integral over the disk of all vectorial field components directed in sight to the observer. If all vectors in the surface elements are nearly parallelly directed, then there would occur a sharp maximum; otherwise, in case of wide-spread distribution of the field over the sphere the maximum would be embedded in a broad profile.

The profile form depends strongly on the local position on the star's globe, the observer looks at. In the course of the star's rotation, the beam viewed by the observer slides along the latitude circle given by the inclination angle i, producing thus a periodically varying geometrical profile with characteristic deviations from symmetry. These characteristics are to be investigated for analyzing and identifying purposes of the line-generating magnitudes.

The asymmetry of the line profiles has serious consequences for the measurement of magnetic fields by the Zeeman-shift between the σ-components of the circularly polarized light. Every observer — so as the authors did three decades ago (see Gerth et al. 1977) — is inclined to measure the line by the spectral position of the maximum or by the best visual correlation of the profile with its mirror image. In that time we could not explain the strange asymmetries of the line profiles seen on the oscilloscope screen. However, with our new knowledge about the all in all *natural* asymmetry of the line profile, we can explain now the uncertainties of our former measurements. They were not wrong at all, because they reveal at least a statistical tendency for the magnitude of the magnetic field strength and its variation, but they suffer all from a very high scatter, which is not only due to the graininess of the photographic plate.

In every case, the center of gravity of the profile gives the best spectral location of the line. Thus, broad lines like those of hydrogen are less affected by the geometrical influence than the sharp metallic lines, because their chare in the profile resulting by convolution is higher. This led to a biased selection of slowly rotating stars, viewed nearly pole-on. However, what was thought as the correct measurement of sharp lines proves as very intriguing and misleading. Nevertheless, the knowledge of the functional relation of the geometry and the line profile could open a further source of information and would improve the correctness and reliability of the measurement of stellar magnetic fields.

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